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Optimal Strategy Analysis of N-Policy M/M/1 Vacation Queueing System with Server Start-Up and Time-Out

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Abstract: *This paper analyzes the optimal strategy N-policy M/M/1 Queueing system with server Start-up and Time-Out. The arrivals are considered according to Poisson Process and service times are follows an exponential distribution. If the server finds queue is empty then the server waits for a fixed time 'C' is called server Time-Out. If one unit has arrived during this fixed time he does service to that unit, otherwise, after expiration of fixed time he takes vacation. He returns from the vacation, if the number of units accumulates 'N' in the queue then the server commences into startup mode then does the service. The distribution of the system size is derived through probability generating functions and obtained other system characteristics. Finally, the expected cost per unit time is considered to determine the optimal operating policy at a minimum cost. The sensitivity analysis has been carried out to examine the effect of different parameters in the system.*
Keywords: *N-Policy, Start-up, Time-out and probability generating functions.*

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I. Introduction

Queueing models with server vacations have several applications in many areas such as in computer networks, in telecommunication systems, in inventory control processes, in manufacturing and in pharmaceutical sectors etc..

In this paper, we consider the optimal strategy of an N-policy M/M/1 queueing system with server *Start-up and Time-Out*. Customers arrive in according to a Poisson process. Waiting customers are served individually. Before the vacation, server waits for a fixed time 'C' is called server Time Out if the queue is empty. If one unit has arrived during this fixed time he does service to that unit. Otherwise, he takes vacation after expiration of fixed time. The server is turned off each time the system empties. The server is turned on as and when the queue length reaches or exceeds N (threshold). Before the service, the system requires a random startup time for pre-service. As soon as the startup period is over the server starts the service to all customers in the queue.

Queueing systems with server vacations, in which server may unavailable for a random period of time 'α' from service station. Arrived customer during the vacation will have to wait until the server returns from vacation. Queueing system with server vacations by using probability generating function technique was first introduced by Levy and Yechiali [5]. Several excellent surveys on vacation models have been done by Doshi [1] [2], and the books by Takagi [8] and Tian and Zhang [9] are devoted to the same subject. In the vacation models that have been analyzed in the literature, server timeouts have not been considered. Oliver C.Ibe and Olubukola A.Isijola[6] studied M/M/1 multiple vacation queueing system with differentiated vacation. Oliver C.Ibe [7] introduced the timeout concept and he derived mean waiting time of vacation queueing systems with server timeout. E.Ramesh Kumar and Y. Praby Loit [3], introduced A Study on Vacation Bulk Queueing Model with setup time and server timeout. Y.Saritha, K.Satish Kumar and K.Chandan [10], derived an expected system length for $M^X/G/1$ Vacation Queueing System with Server Timeout.

In this paper, we consider N-policy vacation queueing systems with server startup and timeouts. The four main objectives for which the analysis has been carried out for the optimal strategy policy are:

i. To determine the steady state probability distribution of the number of customers in the system.
ii. To derive system characteristics such as expected number of customers in the system when the server is in time-out, vacation, in startup and in service states respectively and expected system length.
iii. To formulate the total expected costs function for the system, and determine the optimum value of the parameter N.
iv. To carry out a sensitivity analysis on the optimum value of N and the minimum expected cost through numerical illustrations.

II. Model Description

Let us consider single server queueing system, here customers assumed to arrive in according to a Poisson process with mean arrival rate λ. When the queue size reaches N(≥1) the server will spend a random startup time *t* for pre-service, which is assumed to follow an exponential distribution with mean 1/α, As soon as the period of startup is over, the server begins service to all customers waiting in the queue. Inter arrival and service times follow an exponential distribution with parameter μ. The server commences the service in FIFS discipline. When the server has finished serving a customer and finds the system empty, the server waits for a fixed time C, called server timeout. If a customer arrives before this time expires, the server commences service. If no customer arrives by the end of this time, the server commences vacation. If the number of units accumulates 'N' in the queue during this vacation then the server commences into startup mode and then do service to the waiting units in the queue otherwise it will commence another vacation. The Queueing System with Server time-out and N-policy performance has been defined in below figure.

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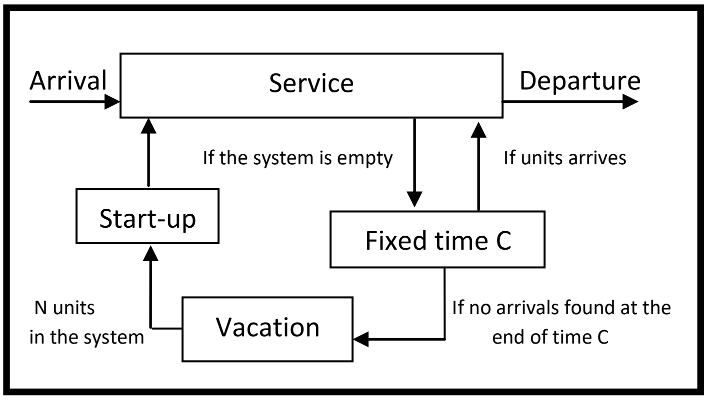

III. Analysis Of The Model

Consider for i=0,1,2,...

P_i^0 =Probabilitythat there are i units in the system when the server is on vacation

P_i^1 =Probabilitythat there are i units in the system when the server is in Start-up

P_i^2 =Probabilitythat there are i units in the system when the server is in Time Out

P_i^3 =Probabilitythat there are i units in the system when the server is in Service

The steady state equations are as follows:

$$\boldsymbol{\lambda P_0^0 = C P_0^2} \qquad (1)$$

$$\boldsymbol{\lambda P_i^0 = \lambda P_{i-1}^0}, \qquad 1 \le i \le N-1, \qquad (2)$$

$$\boldsymbol{(\lambda+\alpha) P_N^1 = \lambda P_{N-1}^0}, \qquad (3)$$

$$\boldsymbol{(\lambda+\alpha) P_i^1 = \lambda P_{i-1}^1}, \qquad \boldsymbol{N+1 \le i \le \infty} \qquad (4)$$

$$\boldsymbol{(\lambda+C) P_0^2 = \mu P_1^3}, \qquad (5)$$

$$\boldsymbol{(\lambda+\mu) P_1^3 = \lambda P_0^2 + \mu P_2^3}, \qquad (6)$$

$$\boldsymbol{(\lambda+\mu) P_i^3 = \lambda P_{i-1}^3 + \mu P_{i+1}^3}, \qquad \boldsymbol{2 \le i \le N-1} \qquad (7)$$

$$\boldsymbol{(\lambda+\mu) P_i^3 = \lambda P_{i-1}^3 + \mu P_{i+1}^3 + \alpha P_i^1}, \qquad \boldsymbol{i \ge N} \qquad (8)$$

Probability generating functions are as follows

$$G_0(z) = \sum_{i=0}^{N-1} P_i^0 z^i \,, G_1(z) = \sum_{i=N}^{\infty} P_i^1 z^i,\ G_2(z) = P_0^2 \ and\ G_3(z) = \sum_{i=1}^{\infty} P_i^3 z^i,$$

From the equations (1), $\boldsymbol{P_0^2 = \frac{\lambda}{C} P_0^0}$, (9)

From the equations (2), $\boldsymbol{(1-z) G_0(z) = (1-z^n) P_0^0}$, (10)

From the equations (3) and (4), $\boldsymbol{[\lambda(1-z)+\alpha] G_1(z) = \lambda z^n P_0^0}$, (11)

From the equations (5), $\boldsymbol{P_1^3 = \frac{\lambda(\lambda+C)}{C\mu} P_0^0}$, (12)

From the equations (6), (7) and (8)

$$\boldsymbol{[\lambda z(1-z) + \mu(z-1)] G_3(z) = \lambda z^2 P_0^2 + \alpha z G_1(z) - \mu z P_1^3}, \qquad (13)$$

From the equations (9) and (13)

$$\boldsymbol{G_0(1) = N P_0^0} = \boldsymbol{\frac{E_0}{E_C}} \qquad (14)$$

$$\boldsymbol{G_1(1) = \frac{\lambda}{\alpha} P_0^0 = \frac{E_1}{E_C}} \qquad (15)$$

$$\boldsymbol{G_2(1) = \frac{\lambda}{C} P_0^0 = \frac{E_2}{E_C}} \qquad (16)$$

$$\boldsymbol{G_3(1) = \frac{\lambda}{\mu-\lambda}\left[N + \frac{\lambda}{\alpha} + \frac{\lambda}{C}\right] P_0^0 = \frac{\lambda}{\mu} = \frac{E_3}{E_C}} \qquad (17)$$

We know that , $G(z)=G_0(z)+G_1(z)+G_2(z)+G_3(z)$ (18)

For z=1, $G(1)=G_0(1)+G_1(1)+G_2(1)+G_3(1)=1$

From the above condition we get, $P_0^0 = \frac{(1-\frac{\lambda}{\mu})}{[N+\frac{\lambda}{\alpha}+\frac{\lambda}{C}]}$ (19) and $\frac{1}{E_C} = \frac{\lambda(1-\frac{\lambda}{\mu})}{[N+\frac{\lambda}{\alpha}+\frac{\lambda}{C}]}$ (20)

IV. Performance Measures

Let L_0, L_1, L_2 and L_3 are the expected lengths for the server being in vacation, startup, Time out and service. By differentiating equation (18) with respect to z with z=1 then we get the following

$$L_0 = \frac{N(N-1)}{2} P_0^0 \qquad (21)$$

$$L_1 = \frac{\lambda}{\alpha}\left(N + \frac{\lambda}{\alpha}\right) P_0^0 \qquad (22)$$

$$L_2 = P_0^2 = 0 \qquad (23)$$

$$L_3 = \frac{\lambda}{\mu-\lambda}\left[\frac{\lambda}{\mu-\lambda}\left(N + \frac{\lambda}{\alpha} + \frac{\lambda}{C}\right) + \frac{\lambda}{C} + N + \frac{\lambda}{\alpha} + \frac{N(N-1)}{2} + \frac{\lambda}{\alpha}\left(N + \frac{\lambda}{\alpha}\right)\right] P_0^0 \qquad (24)$$

Total Expected length is $L=L_0+L_1+L_2+L_3$ (25)

V. Determination Of Optimal N-Policy (N*)

Here, we develop steady state total expected cost function per unit time for the N-policy M/M/1 queueing system with server timeout in which N is a decision variable. With the cost structure being considered, the objective is to determine the operating N-Policy so as to minimize this function. Let

Ch ≡ holding cost per unit time for each customer present in the system,

Cb ≡ cost per unit time for keeping the server on and in operation,

Cm ≡ startup cost per unit time per cycle,

Ct ≡ timeout cost per unit time per cycle,

Cs ≡ setup cost per cycle,

Cv ≡ reward per unit time for the server being on vacation and doing secondary work.

The total expected cost function per unit time is given by

$$T(N)=C_h L + C_b \frac{E_3}{E_C} + C_m \frac{E_1}{E_C} + C_t \frac{E_2}{E_C} + C_s \frac{1}{E_C} - C_v \frac{E_0}{E_C} \qquad (26)$$

From equations (14)-(17), (19)-(20) and (25), determine the optimal operating N by minimizing T(N) in (26) which is equivalent to minimizing

$$T(N)=\left[C_h \left\{\frac{N(N-1)}{2} + \lambda\left(N + \frac{\lambda}{\alpha}\right) + \frac{\lambda}{C} + \left(\frac{\lambda}{\mu-\lambda}\right)^2 \left(N + \frac{\lambda}{\alpha} + \frac{\lambda}{C}\right) + \left(\frac{\lambda}{\mu-\lambda}\right)\left(N + \frac{\lambda}{\alpha} + \frac{\lambda}{C} + \frac{N(N-1)}{2} + \frac{\lambda}{\alpha}\left(N + \frac{\lambda}{\alpha}\right)\right)\right\} + \right.$$
$$Cb\lambda\mu + Cm\lambda\alpha + Ct\lambda C + Cs\lambda - CvN1 - \lambda\mu N + \lambda\alpha + \lambda C$$
(27)

Now we present a procedure to calculate the optimal threshold N*. To get these optimal values we differentiate equation (27) with respect to N and then equate to zero, we get N*. i.e., $\frac{\partial T(N)}{\partial N}=0$ and $\frac{\partial^2 T(N)}{\partial N^2}>0$, then we get the optimal threshold N* for this model which is given by

$$N^* = \sqrt{\left(\frac{2\lambda\alpha\mu + 2\lambda\mu C + C\lambda\alpha}{2C\mu\alpha}\right)^2 - \frac{2\lambda^2\mu - 2\lambda\alpha\mu + 2\lambda^2\alpha - C\lambda\mu + \lambda C^2}{C\mu\alpha} + \frac{2(\mu-\lambda)\left\{C_b\frac{\lambda}{\mu} + C_v\left(\frac{\lambda}{C}+\frac{\lambda}{\alpha}\right) + C_m\frac{\lambda}{\alpha} + C_t\frac{\lambda}{C} + C_s\lambda\right\}}{\mu C_h}} - \frac{2\lambda\alpha\mu + 2\lambda\mu C + C\lambda\alpha}{2C\mu\alpha} \qquad (28)$$

By substituting the value N* in equation (27), then we get the optimum cost T(N*).

VI. Sensitivity Analysis

In order to verify the sensitivity of our analytical results, we perform numerical experiment. The variations of different parameters (both non-monetary and monetary) on the optimal threshold N*, mean number of jobs in the system and minimum expected cost are shown. Parameters for which the model is relatively sensitive would require more attention of researchers, as compared to the parameters for which the model is relatively insensitive or less sensitive. The sensitivity analysis by fixing Non –monetary parameters as $\lambda=1$, $\mu=4$, $\alpha=2$, $C=1$ and monetary parameters as $C_b=300$, $C_m=200$, $C_t=30$, $C_s=500$, $C_v=15$, $C_h=5$ is performed.

Table 1: Effect of variation in Non –monetary parameters λ, μ, α and C on N*, expected system length(L) and optimum cost(T(N*)).

Parameter	Values	N*	L	T(N*)
λ	1	17.42141	8.406344	60.20488
	1.2	18.21067	8.865835	65.08262
	1.4	18.71668	9.199804	68.86219
	1.6	18.9722	9.428698	71.69423
	1.8	18.99715	9.567935	73.68496
μ	4	17.42141	8.406344	60.60488
	9	17.29506	8.142981	60.50238
	14	17.14518	8.02306	60.45273
	19	17.05591	7.958666	60.41133
	24	16.99831	7.918772	60.38232
α	2	17.42141	8.406344	60.20488
	2.5	16.69931	7.995149	58.86354
	3	16.19698	7.710993	57.97046
	3.5	15.82683	7.502603	57.33554
	4	15.54254	7.34312	56.86223
C	1	17.42141	8.406344	60.20488
	1.1	17.3847	8.423541	60.43259
	1.2	17.35372	8.437918	60.62462
	1.3	17.32723	8.450111	60.78875
	1.4	17.30429	8.460576	60.93065

From Table 1, we observe that

- As λ increasing, N*, expected system length (L) and minimum expected cost (T(N*)) are increasing.
- As μ and α increasing N*, expected system length (L) and minimum expected cost (T(N*)) are decreasing.
- As C increasing N* decreasing, expected system length (L) and minimum expected cost (T(N*)) are increasing.

Table 2: Effect of variation in monetary parameters C_b, C_m, C_t, C, C_v and C_h **on N*, expected system length(L) and optimum cost(T(N*)).**

Parameter	Values	N*	L	T(N*)
$\mathbf{C_b}$	300	17.42141	8.406344	60.20488
	350	17.90742	8.64654	61.19904
	400	18.38164	8.881036	62.19092
	450	18.84487	9.110218	63.17921
	500	19.29785	9.334432	64.16292
$\mathbf{C_m}$	200	17.42141	8.406344	60.20488
	220	17.81119	8.59897	61.00034
	240	18.19332	8.787896	61.79453
	260	18.56821	8.973326	62.58673
	280	18.93627	9.155448	63.37635
$\mathbf{C_t}$	30	17.42141	8.406344	60.20488
	38	18.41909	8.899561	61.66769
	46	19.36942	9.369866	63.16982
	54	20.27855	9.820183	64.69314
	62	21.15143	10.25286	66.22551
$\mathbf{C_s}$	500	17.42141	8.406344	60.20488
	510	17.81119	8.59897	61.38872
	520	18.19332	8.787896	62.55621
	530	18.56821	8.973326	63.7079
	540	18.93627	9.155448	64.84432
$\mathbf{C_v}$	15	17.42141	8.406344	60.20488
	20	17.86899	8.62754	57.6019
	25	18.30652	8.843883	54.98682
	30	18.73466	9.05568	52.35958
	35	19.15397	9.263204	49.72018
$\mathbf{C_h}$	5	17.42141	8.406344	60.20488
	8	17.74597	8.56673	87.69867
	11	18.10021	8.741854	116.28
	14	18.46048	8.920032	145.9527
	17	18.82007	9.097945	176.697

From Table 2, we observe that

- As $\mathbf{C_b}$, $\mathbf{C_s}$, $\mathbf{C_t}$, $\mathbf{C_m}$,$\mathbf{C_h}$ increasing, N*, expected system length (L) and minimum expected cost (T(N*)) are increasing.
- As $\mathbf{C_v}$ increasing, N*, expected system length (L) and minimum expected cost (T(N*)) are decreasing.

VII. Conclusion

In this model, we have derived explicit expressions for the system length of a queueing system whenever server being in time-out, vacation, startup, and in service. Sensitivity analysis made for and numerical values are presented for the different values of monetary and non-monetary parameters to illustrate the validity of the proposed model.

References

[1] Doshi, B.T. (1986) Queueing Systems with Vacations, a Survey. *Queueing Systems*, **1**,29-66.
[2] Doshi, B.T. (1990) Single-Server Queues with Vacations. In: Takagi, H., Ed., *Stochastic Communications Systems*, Elsevier, Amsterdam.
[3] E.Ramesh Kumar and Y. Praby Loit (2016), A Study on Vacation Bulk Queueing Model with setup time and server timeout. IJCMS, 5(12), 81-89.
[4] Kleinrock, L. (1975) Queueing Systems, Volume 1: Theory. John Wiley & Sons, New York.
[5] Levy, Y. and Yechiali, U. (1975) Utilization of Idle Time in an M/G/1 Queueing System. *Management Science*, **22**, 202-211. http://dx.doi.org/10.1287/mnsc.22.2.202
[6] Oliver C.Ibe and Olubukola A.Isijola (2014) M/M/1 multiple vacation queueing system with differentiated vacations, Modeling and Simulation in Engineering vol 2014(2014), Article ID 158247,6 pages http://dx.doi.org/10.1155/2014/158247.
[7] Oliver C. Ibe (2015) M/G/1 Vacation Queueing Systems with Server Timeout, American Journal, 5, 77-88. http://www.scrip.org/journal/ajor.
[8] Takagi, H. (1991) Queueing Analysis: A Foundation of Performance Analysis, Volume 1: Vacation and Priority Systems, Part 1. Elsevier Science Publishers B.V., Amsterdam.
[9] Tian, N. and Zhang, G. (2006) Vacation Queueing Models: Theory and Applications. Springer-Verlag, New York.
[10] Y.Saritha, K.Satish Kumar and K.Chandan, "M^X/G/1 Vacation Queueing system with server timeout", International Journal of Statistics and Applied Mathematics, Vol. 2, No. 5, (2017), 131-135.

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