## **Optimum Design of Mechanism - A Review**

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This paper reviews the methods developed for balancing of the planar mechanisms and synthesizing the link shapes. The methods used for partial force and moment balance including the optimization methods as well as the methods used for the link shape synthesis are reviewed. Several review papers such as Kamenskii (1968a), Lowen and Berkof (1968), Lowen et al. (1983), Kochev (2000), Arakelian and Smith (2005), Wijk et al. (2009), and Arakelian and Briot (2015) throw light on the quantum of work carried out on the dynamic balancing of themechanisms.

Date of Submission: 06-11-2017 Date of acceptance: 17-11-2017

#### I. Partial Shaking Force and Shaking MomentBalancing

Instead of complete balancing of shaking force and shaking moment independently, minimization of them simultaneously is more useful from the design point of view. The optimization methods used to simultaneously minimize the shaking force and shaking moment in planar mechanism can be categorised as:

#### 1.1 Method of harmonic balancing

A method based on the harmonic analysis is used to balance the harmonics of the shaking forces and shaking moments in which the forces and moments are formulated using Gaussian least-square formulation and Fourier series (Norton, 2011). Han (1967) presented a least-square approach to balance the complex mechanisms in which a counterweight connected to the input shaft results in first harmonic balancing of the inertia induced forces and moments. Stevensen (1973) presented a method for the complete balancing of a harmonic unbalance of the machine, including inertia forces, moments of the inertia forces, and inertia torques, utilizing six weights on three shafts in the machine parallel to the coordinate axes and rotating at the speed of the harmonic. Similarly, Tsai (1984) used two Oldham couplings to balance second harmonics of the shaking force and shaking moment. This type of balancer runs at the primary speed of the machine whereas the Lanchester balancer runs at double of the primary speed to get the same balancing effect (Hsieh and Tsai, 2009). A method was developed based on the harmonic balancing to find the region boundaries to locate the additional shafts by Davies and Niu (1994). Arakelian and Dahan (2001) presented a method to achieve the first harmonic balancing of the shaking moment by using the counterweight connected to the input shaft axis (Fig. 1).

#### **1.2** Extension of method of linearly independent vectors

Based on the fact that the in a force balanced mechanism, the shaking moment reduces to a pure torque, Berkof and Lowen (1971a, b) developed the theory of shaking moment optimization. In this method, RMS value of the shaking moment is minimized for a force balanced in-line planar four-bar mechanism. The link mass distribution ratios as the functions of the link length ratios are defined as the design variables. This method was the extension of previously developed force balancing method by Berkof and Lowen(1969).

Fig. 1 First harmonic balancing of the shaking moment in planar four-bar mechanism (Arakelian and



The angular momentum principle is used to define the shaking moment of a force balanced four-bar mechanism. It was shown that the shaking moment cannot be completely balanced without using additional links, but a partial moment balance may be obtained by the desired internal mass rearrangement. Carson and Stephens (1978) extended this method by considering feasible limits for the link parameters. The optimum mass distribution is found for the mechanism links to balance it, and the kinematic design criterion is satisfied by fixing the link length ratios. Similarly, Haines (1981) optimized the RMS values of the shaking moment and the driving torque for a force balanced mechanism by constraining the link parameters within the physicallimits. Tepper and Lowen (1975) presented a method for partial force balancing of four-bar mechanisms, which allows control over the increase in ground bearing forces. They presented a trade-off method, wherein the RMS shaking force of constant input speed, a general four-bar mechanism is optimized while the increase of the RMS ground bearing forces is limited using Lagrange multiplier formulation. They presented an optimization method that represent feasible trade-off techniques for optimizing the RMS shaking force of a four-bar mechanism while keeping the RMS ground bearing forces lower than those of the fully force balancedmechanism.

#### **1.3** Optimum design of counterweights

A partial force balancing method for a planar four-bar mechanism was developed by Tricamo and Lowen (1983a, b) which allows the prescribed maximum shaking force. The simultaneous minimization of shaking moment, input torque and bearing forces is achieved by using the three-counterweight technique (Fig. 2). This method determines the parameters of the three counterweights that must be attached to the input, coupler, and output links, respectively.



Fig. 2 Balancing of planar four-bar mechanism using three counterweights (Tricamo and Lowen, 1983a,b)

Demeulenaere et al. (2004, 2006, and 2010) developed a convex optimization technique to determine the counterweight parameters to balance the planar mechanisms. This technique is used to design point-mass counterweights and minimum-inertia counterweights for the planar mechanisms. The counterweight parameters are chosen as the design variables while the upper and lower limits on mass and center of mass coordinates are presented as the convex constraint functions. The ratio of RMS values of the optimized dynamic force to the original dynamic force known as *balancing effect index* is defined as the objective function. Verschuure (2009) extended this method and introduced a set of auxiliary parameters,  $\mu$ - parameters that are a function of the classical mass parameters to describe a counterweight. Due to the introduction of these parameters, an analytical derivation of the mechanism kinematics and dynamics is not necessary for this method (Fig.3).



Fig. 3 Partial balancing of planar ten-bar mechanism using sector-type counterweights (Verschuure, 2009)

Farmani et al. (2011) formulated the mechanism balancing problem as a multi- objective optimization problem based on the concepts of inertia counterweights and physical pendulum developed by Berkof (1973) which completely balance all linear and rotary mass effects. The thickness of counterweights and disks for both input and output links of a planar four-bar mechanism are considered as the objective functions and minimized using Particle Swarm Optimization (PSO) and Genetic Algorithm (GA). The multiple optimum solutions were found for the optimization problem and then a fuzzy decision maker was used to select the best solution.

#### 1.4 Optimum mass distribution using equimomental system of point-masses

Gill and Freudenstein (1983a, b) developed a design procedure for the optimum mass distribution of the links of high-speed spherical four-bar mechanisms. This analysis includes a quadratic-programming technique that allows an optimum trade-off between shaking forces, shaking moments, bearing reactions, and input-torque fluctuation. They replaced the mass distribution of a rigid link with a fixed point by four equivalent point masses to simplify inertia-force calculations. Rahman (1996) extended this method to the general spatial mechanisms. The previous method was limited to one particular type i.e. spherical mechanism. Lee and Cheng (1984) used a two point-mass model for combined balancing of shaking force, shaking moment, and torque by developing the optimality criterion using Lagrangian and Newtonian formulations. They formulated an optimization problem for the mechanism balancing and solved the same by Heuristic Optimization Technique of Lee and Freudenstein (1976a, b). This method presents the trade-offs among shaking force, shaking moment, input-torque fluctuations and bearing reactions by redistributing the link masses and adding counterweights. Thus, the optimum balancing of the mechanism can be achieved in the design stage. The two-point mass model developed by Wiederich and Roth (1976) for momentum conservation was adopted for the modeling of the inertial properties of themechanism.

Chaudhary and Saha (2007, 2008 and 2009) solved the mechanism balancing problem as an optimization problem by finding the optimum mass distribution of the mechanism links along with constraining the mechanism parameters within the feasible limits. Thus, the shaking force and shaking moment are minimized simultaneously by optimizing links inertial properties, i.e., mass, location of center of mass and moment of inertia. In this method, the inertia properties of the mechanism are represented by the equimomental point-mass systems that are dynamically equivalent to the rigid moving links of the mechanisms. The equimomental point- mass systems are used to identify the design variables and to formulate the constraints for the optimization problem formulation. They proposed three equimomental point- masses for each link of the planar mechanisms (Chaudhary and Saha, 2006). To determine the shaking force and the shaking moment, the equimomental point-masses parameters are used to formulate the dynamic equations of motion for the mechanisms. Anoptimizationproblemisformulatedforfindingtheoptimummass distribution of the mechanism links that minimizes the shaking force and shaking moment. The point-mass parameters are defined as the design variables and limits on links masses, and inertias are presented as the constraints for the optimization problem. The objective function is defined as the weighted sum of the dimensionless RMS values of the shaking force and shaking moment calculated using Newton-Euler equations of motions.

#### **1.5** Optimum mass distribution to reduce effect of jointclearances



Fig. 4 Planar four-bar mechanism with joint clearance (Erkaya and Uzmay 2009)

For the mechanisms with a clearance at joints as shown in Fig. 4, the changes in the direction and magnitude of the joint forces produce the vibrations and thus considered as the critical design parameters (Zhe and Shixian, 1992). An optimization method is developed to minimize the adverse dynamic effects produced due to the joint clearances (Park and Kwak, 1987). In this method, the magnitude and location of the counterweight added to the mechanism are selected as the design variables. Similarly, Feng et al. (2002) reduced the joint forces by finding the optimum mass distribution for the link masses. The change in direction and amplitude of the joint forces is calculated using the Lagrange's equations of motion and presented as the objective function whereas the inertial parameters of the moving links, i.e., the mass, the location of mass center and the moment of inertia are taken as the design variables. Erkaya and Uzmay (2008, 2009) presented an optimization method to reduce the effect of joint clearance on the shaking force and shaking moment in the planar mechanism. The weighted sum of differences of force and moment values with and without the clearance is defined as the objective function to be minimized. The link lengths and their center of mass locations are defined as the design variables with proper lower and upper limits to solve this optimizationproblem.

#### 1.6 Non-linear constraint optimization using natural orthogonal complement method

An optimization method was developed to balance a five-bar mechanism using natural orthogonal complement dynamic modeling (Ilia and Sinatra, 2007, 2009). The shaking force is minimized through conventional optimization method, i.e., non-linear constraint optimization. The center of mass parameters of moving links were chosen as the design variables while the natural orthogonal complement method was used for dynamic analysis of the mechanism.

#### **1.7** Mixed mass redistributionmethod

The mixed mass redistribution method combining the principles of the mass redistribution and addition of counterweights is applied for reducing the shaking force and shaking moment in the mechanisms (Guo et al., 2000; Feng et al., 2000). The mass, moment of inertia and location of mass center of the moving links along with the mass and mass center location of the counterweights were taken as the design variables.

#### 1.8 Balancing through integrating kinematic and dynamiccharacteristics

Yan and Soong (2001 and 2004) presented an optimization method integrating kinematic synthesis, dynamic balancing design, and the design of the input link's motion trajectory. The trajectory of the crank displacement is represented by a Bezier curve while the kinematic and dynamic properties of the mechanism are included in the objective function using appropriate weighting factors.

Thus, the kinematic design requirements are satisfied along with the reduction in the shaking force and shaking moment for the mechanism. This method was further extended using the bi-material moving links (Soong and Hsu, 2007). The moving links of the mechanism are designed by joining two different materials with optimum link length proportions as shown in Fig. 5.

#### 1.9 Sensitivityanalysis

Sensitivity analysis is the study of the uncertainty in the output of a mathematicalmodel to different sources of uncertainty in its inputs. Li (1998) presented a method to analyse the sensitivity of the shaking force and shaking moment to the design variables for a planar mechanism. The objective function includes shaking force, shaking moment and the sensitivity terms such as the ratio of shaking moment to mass center distance of links; is minimized by optimizing the inertial parameters of the links treated as the design variables. Alici and Shirinzadeh (2006) formulated an optimization problem to dynamically balance the planar parallel manipulators. In this method, the objective function is defined as the sum-squared values of shaking moment, driving torques, bearing forces and the angular momentum deviation.

The constraints are imposed on the geometric as well as the inertial properties along with the full force balancing condition for themechanism.



**Fig. 5** Balancing of four-bar mechanism by designing bi-material moving links (Soong and Hsu, 2007)

In another method, Erkaya (2013) formulated the objective function by adding the sub-components of shaking force and moment and analysed the sensitivity of the optimum result to weighting factors used in the objective function. The kinematic and dynamic parameters such as link length, structural angle, mass, inertia and center of mass location are defined as the design variables. In addition to the methods discussed above the forces and moments transmitting to the engine mounts due to reciprocating parts and gas pressure in a four-stroke seven-cylinder marine engine are reduced by optimizing the crank angles (Park et al., 2007). The steepest decent method with golden section search was used for the optimization.

### II. Methods for Link Shape Synthesis

After obtaining optimum inertial properties for the balanced mechanism, links shapes are to be decided to carry various loads acting on them. Several methods are suggested in the literature to find the mechanism link shapes for specified inertial properties that require an initial design domain to start with.

# The methods for link shape synthesis available in the literature may be categorised as: 2.1Small element superposingmethod

Feng et al. (2002) developed a Small Element Superposing Method (SESM) to find link shapes which discretize the initial assumed shape into small mass elements and locate them systematically along the link length (Fig. 6). The design equations corresponding to the optimized mass, center of mass location and inertia are satisfied to form the link by superposing these small mass elements.

#### 2.2Voxel-baseddiscretization

In the convex optimization method developed by Demeulenaere (2004a, b, 2006, 2010) designing point-mass counterweights and minimum-inertia counterweights was reformulated as a convex nonlinear optimization problem. This method was used to optimally design the counterweights used for the balancing purpose.



Fig. 6 Small element superposing method for finding link shape (Feng et al., 2002)

For planar mechanisms, a particular counterweight shape is enforced through the additional constraints. The counterweight shapes considered were point-mass, minimum-inertia counterweights and sector-type counterweights. Based on this approach Verschuure (2008a, b, 2009) used a voxel-based discretization to describe the final counterweight shape, inspired by the typical discretizations used in the area of topologyoptimization.

#### 2.3Evolutionary Structural Optimization method

The Evolutionary Structural Optimization (ESO) method is used to optimize the rotating machinery shaft's shape by gradually removing ineffective material from the design domain (Xie and Steven, 1993, 1996; Kim et. al., 2002). This method executes the Finite Element Analysis (FEA) with finite size elements in each iteration to find the optimum shape.

### 2.4Gradient-basedmethod

In this method, the link shapes are synthesized by maximizing external work done by a given external force considering total volume of all links as the constraint function (Azegami et al., 2013). The external work done for the mechanism defined as the objective function satisfies the requirement of the kinetic energy of the mechanism for an assigned time interval. The initial design domain for the link is considered as the design variable and is optimized to get optimum link shape (Fig. 7).



Optimumshapes

Fig. 7 Optimum shape for piston-crank mechanism using gradient-based method (Azegami et al., 2013)

#### 2.5Topology Optimization Method

The link shapes are also found through the topology optimization based on parametric curves (Xu and Ananthasuresh, 2003) and non-intersecting closed polygons (Yoganand and Sen, 2011). In this approach, the vertices of the control polygon that define parametric curves for links are used to identify the design space. Based on FEA, this formulation considers the constraints to avoid self-intersections and intersections with other segments. It also ensures the optimum utilization of the available space and material. The required inertial properties of the design domain are achieved through optimizing the vertices of the control polygon as shown in



Fig. 8 Geometry synthesis through topology optimization (Yoganand and Sen, 2011)

#### 2.6Shapes For Specific Path Andmotion

The link shapes for the interference-free motion are found by identifying feasible material domain associated with the link geometries (Sen et al., 2004). Similarly, the mechanism's dimensional synthesis to generate specified path or motion based on graphical and analytical techniques can also be used for shape optimization (Sandor and Erdman, 1984; Freudenstein, 2010).

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International Journal of Engineering Science Invention (IJESI) is UGC approved Journal with Sl. No. 3822, Journal no. 43302.

Dr. Kailash Chaudhary Optimum Design of Mechanism - A Review." International Journal of Engineering Science Invention(IJESI), vol. 6, no. 11, 2017, pp. 01-09.