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A Two Warehouse Inventory Model Under Inflation With Variable Deterioration And Partially Backordering

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I. INTRODUCTION

In general, when suppliers provide price discounts for bulk purchases or when the item under consideration is a seasonal product such as the output of harvest, the manager may purchase more goods than can be stored in a rented warehouse (RW). Further, the inventory costs (including holding cost and deterioration cost) in RW are usually higher than those in OW due to additional cost of maintenance, material handling etc. For example, the RW like "central warehousing facility" generally provides better preserving facility than the OW resulting in a lower deterioration rate for the goods. To reduce the inventory costs, it will be economical to consume the goods at the earliest. As a result, the firm stores goods in OW before RW, but clears the stocks in RW before OW In many real life situations, the practical experiences reveal that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable commodities or high-tech products with short product life cycle. The longer the waiting time is the smaller the backlogging rate would be according to such phenomenon, taking the backlogging rate into account is necessary.

Various researchers have discussed two-warehouse inventory system. In this system, it is assumed that the holding cost in RW is greater than that in OW. Hence, items in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero and than items in OW are released. Pakkla and Achary [1991] developed a two-warehouse probabilistic order level inventory model for deteriorating items. Pakkla and Achary [1992] developed a deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. These models assumed time as a continuous variable. Pakkla and Achary [1992] also developed a discrete-in-time model for deteriorating items with two warehouses. Benkherouf [1997] relax the assumptions of fixed cycle length and specified quantity to be stocked in OW. He found the optimal schedule that minimizes the total cost per unit time in a cycle for an arbitrary demand rate function where the cycles are assumed to be a regenerative process. Bhunia and Maity [1998] analyzed a deterministic inventory model with different levels of item deterioration in both warehouses. Zhou [2003] developed a deterministic model with multiple warehouses possessing limited storage capacity. The demand rate is a function of time. The model allows shortage in OW. Yang [2004] considered a two-warehouse inventory models for constant deteriorating items with constant demand rate and complete shortages under inflation. Most of the classical inventory models assumed the utility of the inventory remains constant during their storage period. But in a real life, deterioration does occur in storage. In the past few decades, many researchers on replenishment problem for deteriorating items such as volatile liquids, medicines, seasonal food and others has been studied. Dave and Patel [1981] considered an inventory model for deteriorating items with time-proportional demand when shortages were prohibited. Sachan [1984] then extended the model to allow for shortages. Later, Hariga [1996] generalized the demand pattern to any log concave function. Teng et al. [1999] further generalized the demand function to include any non-negative, continuous function that fluctuates with time. Recently, Goyal and Giri [2001] presented a review of the inventory literature for deteriorating items since the early 1990`s.

In addition, due to high inflation rate, the effects of inflation and time value of money are vital in practical environment, especially in the developing national market. To relax the assumption of no inflationary effects on costs, Buzacott [1975] and Misra [1975] simultaneously developed EOQ models with constant demand and a single inflation rate for all associated costs. Bierman and Thomas [1977] then proposed an inflation model for the EOQ that incorporated the time value of money. Again under the assumption of constant demand, Misra [1979] first provided different inflation rates for various costs associated with an inventory system. Base et al. [1995] developed the EOQ inventory model under inflation rate and time discounting.

In this study, a two-warehouse inventory model with partial backordering and Weibull distribution deterioration is developed. We consider inflation and apply the discounted cash flow in problem analysis. The inventory system under consideration does not have sufficient space to accommodate the on-hand inventory. In such situation, we are stored at own warehouse (OW) and excess inventory is required to be kept in Rental warehouse (RW). The holding costs at RW are higher as compared to OW. When only rented or own warehouse is considered, the present value of the total relevant cost is higher than the case when two-warehouse is considered. Most researchers on inventory models do not consider simultaneously, varying rate of deteriorating, inflation and partial backordering. Since these phenomena are not uncommon in real life, we incorporate them in our problem development. This study takes into account of inflation for problem analysis. The optimization framework is presented to derive the optimal replenishment policy that minimizes the total cost. We show that the total cost of the system is influenced by the deterioration rate, the inflation rate and the backordering rate.

Assumptions And Notation

The mathematical model in this paper is developed based on the following assumptions:

(1) Demand rate is exponential and is equal to (ae^{bt}), where 'a' and 'b' are constants.
(2) Shortages are allowed and they are partially backlogged.
(3) Deterioration of the item follows a two-parameter Weibull distribution.
(4) Deterioration occurs as soon as items are received into items during the period under consideration.
(5) Carrying cost applies to good units only.
(6) Continuous cost compounding is used throughout the analysis of inventory.
(7) There is no replacement or repair of deteriorating items.
(8) The holding costs in RW are higher than those in OW.
(9) The OW has a fixed capacity of W units and the RW has unlimited capacity.
(10) Lead-time is zero and initial inventory level is zero.
(11) The replenishment rate is infinite.

The following notations are used throughout the paper:

d Demand rate (units/unit time)
W Capacity of OW
α Scale parameter of the deterioration rate in OW
β Shape parameter of the deterioration rate in OW
g Scale parameter of the deterioration rate in RW, a>g
h Shape parameter of the deterioration rate in RW
r Inflation rate
B Fraction of the demand backordered during the stock out period
C_1 Ordering cost per order ($/order)
C_{02} Holding cost per unit time in OW ($/unit/unit time)
C_{r2} Holding cost per unit per unit time in RW ($/unit/unit time), $C_{r2}>C_{02}$
C_3 Shortage cost per unit time ($/unit/unit time)
C_4 Shortage cost for lost sales per unit ($/unit)
C Item cost per unit ($/unit)
Q_0 The order quantity in OW
Q_r The order quantity in RW
I_r Maximum inventory level in RW
T_2 Time with positive inventory in RW
T_1+T_2 Time with positive inventory in OW
T_3 Time when shortage occurs in OW
T Length of the cycle, $T=T_1+T_2+T_3$
$I_{0i}(t_i)$ Inventory level in OW at time t_i, $0\le t_i\le T_i$, I=1,2,3
$I_r(t_1)$ Inventory level in RW at time t_1, $0\le t_1\le T_1$
TUC The present value of the total relevant cost per unit time
T Time of deterioration, t>0
f(t) Probability density function of product life (p.d.f.)
F(t) Cumulative distribution functions of product life (c.d.f.)
R(t) Reliability
Z(f) Instantaneous rate of deterioration

From the reliability theory, one has R(t)=1-F(t), Z(t)=f(t)/R(t) and R(0)=1. The product life t is assumed to follow a two-parameter Weibull distribution. We assume x as scale parameter, y as shape parameter and x, y>0. One has

$$f(t) = xyt^{y-1}e^{-xt^y}$$

$$F(t) = \int_{-\infty}^{t} f(t)dt = 1 - e^{-xt^y}$$

$$R(t)=1-F(t)=e^{-xt^{y}}$$

$$Z(t)=\frac{f(t)}{R(t)}=\frac{xyt^{y-1}e^{-xt^{y}}}{e^{-xt^{y}}}=xyt^{y-1}$$

The instantaneous rate of deterioration $Z(t)=xyt^{y-1}$ is used in the following model development.
When y>1, deteriorating rate increases with time. When y<1, deteriorating rate decreases with time. And when y=1, deteriorating rate is constant. The two-parameter Weibull distribution reduces to the exponential distribution.

II. Model Development

The OW inventory system in Fig. 1 can be divided into three phases depicted by T_1 to T_3. For each replenishment, a portion of the replenished quantity is used to backorder shortage, while the rest enters the system. W units of items are stored in the OW and the rest is dispatched to the RW. The RW is therefore utilized only after OW is full, but stocks in RW are dispatched first. Stock in the RW depletes due to demand and deterioration until it reaches zero. During that time, the inventory in OW depletes due to the combined effect of demand and deterioration during time T_2. During the time T_3, both warehouses are empty, and part of the shortage is backordered in the next replenishment.
Inventory level

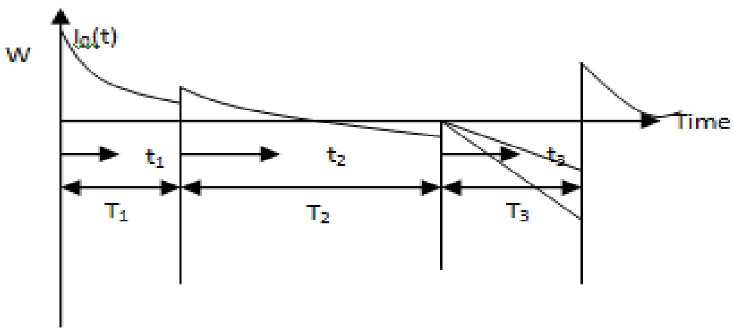

Fig 1. Graphical representation for the OW inventory system

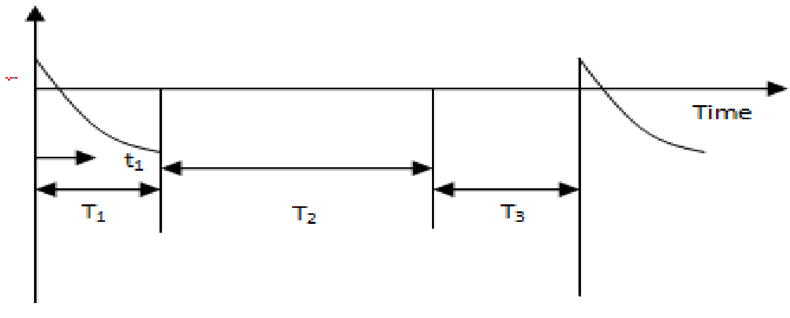

Fig 2. Graphical representation for the RW inventory system

OW inventory system can be represented by the following differential equations:

$$\frac{dI_{01}(t_1)}{dt_1}=-\alpha\beta\, t_1^{\beta-1}I_{01}(t_1) \qquad 0\le t_1\le T_1 \qquad \ldots(1)$$

$$\frac{dI_0(t_2)}{dt_2}=-ae^{bt_2}-\alpha\beta\, t_2^{\beta-1}I_{02}(t_2), \qquad 0\le t_2\le T_2 \qquad \ldots(2)$$

$$\frac{dI_{03}(t_3)}{dt_3} = -\beta\, a e^{bt_3} \qquad 0 \le t_3 \le T_3 \qquad \ldots(3)$$

The first order differential equations can be solved using the boundary conditions, $I_{01}(0)=W$, $I_{01}(T_1)= I_{02}(0)= We^{-\alpha T_1^{\beta}}$ and $I_{03}(0)=0$, one has

$$I_{01}(t_1) = We^{-\alpha T_1^{\beta}} \qquad 0 \le t_1 \le T_1 \qquad \ldots(4)$$

$$I_{02}(t_2) = \frac{We^{-\alpha T_1^{\beta}} - a\int_0^{t_2} e^{bu+\alpha u^{\beta}}\,du}{e^{\alpha t_2^{\beta}}} \qquad \ldots(5)$$

$$\therefore \qquad I_{03}(t_3) = -\frac{Ba}{b}e^{bt_3} + \frac{Ba}{b} \Rightarrow \frac{Ba}{b}\left[1-e^{bt_3}\right] \qquad \ldots(6)$$

The RW inventory system can be represented by the following differential equations:

$$\frac{dI_r(t_1)}{dt_1} = -ae^{bt_1} - ght_1^{h-1} I_r(t_1) \qquad 0 \le t_1 \le T_1 \qquad \ldots(7)$$

The first order differential equation can be solved using the boundary condition, $I_r(0)=I_r$, one has

$$I_r(t_1) = \frac{I_r - a\int_0^{t_1} e^{bu+gu^h}\,du}{e^{gt_1^h}} \qquad \ldots.(8)$$

where $I_r = a\int_0^{T_1} e^{bu+gu^h}\,du$

$$= a\int_0^{T_1}(1+bu+\alpha u^{\beta})\,du = a\left[u + \frac{bu^2}{2} + \frac{g}{h+1}u^{h+1}\right]_0^{T_1} = a\left(T_1 + \frac{b}{2}T_1^2 + \frac{g}{h+1}T_1^{h+1}\right) \qquad \ldots(9)$$

(1) From fig 1, replenishment is made at $t_1=0$, the present worth ordering cost is
$OR=C_1$

(2) Inventory occurs during T_1 and T_2 time periods. The OW present worth inventory cost is

$$HD_0 = C_{02}\left\{\int_0^{T_1} I_{01}(t_1)\,e^{-rt_1}dt_1 + \int_0^{T_2} I_{02}(t_2)\,e^{-r(T_1+t_2)}\,dt_2\right\}$$

$$= C_{02}\left\{\int_0^{T_1}\left(We^{-\alpha T_1^{\beta}}\right)e^{-rt_1}dt_1 + \int_0^{T_2}\left(\frac{We^{-\alpha T_1^{\beta}} - a\int_0^{t_2} e^{bu+\alpha u^{\beta}}\,du}{e^{rt_2^{\beta}}}\right)e^{-r(T_1+t_2)}\,dt_2\right\}$$

$$= C_{02}\left\{\int_0^{T_1} W\left[\sum_{m=0}^{\infty}\frac{(-\alpha T_1^{\beta})^m}{m!}\right]\left[\sum_{m=0}^{\infty}\frac{(-r t_1)^m}{m!}\right]dt_1\right.$$

$$\left.+\int_0^{T_2}\left(\frac{W\left[\sum_{m=0}^{\infty}\frac{(-\alpha T_1^{\beta})^m}{m!}\right]-a\left(t_2+\frac{b}{2}t_2^2+\frac{\alpha}{\beta+1}t_2^{\beta+1}\right)}{\left[\sum_{m=0}^{\infty}\frac{(-\alpha t_2^{\beta})^m}{m!}\right]}\right)\left[\sum_{m=0}^{\infty}\frac{(-r(T_1+t_2))^m}{m!}\right]dt_2\right\}$$

$$= C_{02}W\left(T_1-\frac{\alpha}{\beta+1}T_1^{\beta+1}-\frac{r}{2}T_1^2\right)+C_{02}\left[W(1-\alpha T_1^{\beta}-rT_1)T_2+(-Wr-a+arT_1)\frac{T_2^2}{2}\right.$$

$$\left.+\left(ar-\frac{ab}{2}+\frac{ab\alpha}{2}\right)\frac{T_2^3}{3}+\frac{abr}{8}T_2^4-\frac{W\alpha}{\beta+1}T_2^{\beta+1}-\frac{a\alpha T_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\alpha T_2^{\beta+2}}{\beta+2}+\frac{ab\alpha}{2(\beta+3)}T_2^{\beta+3}\right]$$

$$= C_0W\left(T_1-\frac{\alpha}{\beta+1}T_1^{\beta+1}-\frac{r}{2}T_1^2\right)+C_{02}\left[W(1-\alpha T_1^{\beta}-rT_1)T_2+(-W-a+arT_1)\frac{T_2^2}{2}\right.$$

$$\left.+\left(ar-\frac{ab}{2}+\frac{ab\alpha}{2}\right)\frac{T_2^3}{3}+\frac{abr}{8}T_2^4-\frac{W\alpha}{\beta+1}T_2^{\beta+1}+\frac{a\alpha\beta}{(\beta+1)(\beta+2)}T_2^{\beta+2}+\frac{ab\alpha}{2(\beta+3)}T_2^{\beta+3}\right] \quad \text{…….(10)}$$

(3) Shortages occur during T_3 time period. The OW present worth shortage cost is

$$\mathbf{SH} = \mathbf{C}_3\left\{\int_0^{T_3}\left[-I_3(t)\right]e^{-r(T_1+T_2+t_3)}\,dt_3\right\}$$

$$= C_3\left\{\int_0^{T_3}\left[-\frac{Ba}{b}(1-e^{bt_3})\right]\left[\sum_{m=0}^{\infty}\frac{(-r(T_1+T_2+t_3))^m}{m!}\right]dt_3\right\}$$

$$= C_3\left\{\int_0^{T_3}\frac{Ba}{b}(e^{bt_3}-1)\left[(1-r(T_1+T_2+t_3))\right]dt_3\right\}$$

$$= C_3Ba\left\{\int_0^{T_3}t_3\left[(1-r(T_1+T_2+t_3))\right]dt_3\right\}$$

$$= C_3Ba\left\{\int_0^{T_3}\left[(1-r(T_1+T_2))t_3-rt_3^2\right]dt_3\right\}$$

$$= C_3 Ba\left[(1-rT_1-T_2)\frac{T_3^2}{2}-\frac{rT_3^3}{3}\right] \qquad \ldots(11)$$

(4) Lost sales occur during T_3 time period. The OW present worth lost sale cost is

$$LS = C_4\left\{\int_0^{T_3}(1-B)ae^{bt_3}e^{-r(T_1+T_2+t_3)}dt_3\right\} = C_4(1-B)a\left\{\int_0^{T_3}e^{bt_3}e^{-r(T_1+T_2+t_3)}dt_3\right\}$$

$$= C_4(1-B)a\left\{\int_0^{T_3}\left[1-rT_1-rT_2-rt_3-brT_1t_3-brT_2t_3-brt_3^2\right]dt_3\right\}$$

$$= C_4(1-B)a\left[T_3-rT_1T_3-rT_2T_3-\frac{rT_3^2}{2}-\frac{brT_1T_3^2}{2}-\frac{brT_2T_3^2}{2}-\frac{brT_3^3}{3}\right] \qquad \ldots(12)$$

(5) Replenishment occur at t=0 and $t=T_1+T_2+T_3=T$. The item cost includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

$$IT_0 = C\left(W + Ba\,e^{bT_3}T_3\,e^{-r(T_1+T_2+T_3)}\right)$$

$$= C\left[W+Ba\left(\sum_{m=0}^{\infty}\frac{(bT_3)^m}{m!}\right)T_3\left(\sum_{m=0}^{\infty}\frac{(-r(T_1+T_2+T_3))^m}{m!}\right)\right]$$

$$= C\left\{W+BaT_3\left[1-r(T_1+T_2+T_3)+bT_3-brT_3(T_1+T_2+T_3)\right]\right\} \qquad \ldots(13)$$

(6) From fig. 2, inventory occurs during T_1 time periods. The RW present worth inventory cost is

$$HD_r = C_{r2}\left\{\int_0^{T_1}I_r(t_1)e^{-rt_1}dt_1\right\} = C_{r2}\left\{\int_0^{t_1}\left(\frac{I_r - a\int_0^{t_1}e^{bu+gu^h}du}{e^{gt_1^h}}\right)e^{-rt_1}dt_1\right\}$$

$$= C_{r2}\left\{\int_0^{T_1}\left[a\left(T_1+\frac{b}{2}T_1^2+\frac{g}{(h+1)}T_1^{h+1}\right)-a\left(t_1+\frac{b}{2}t_1^2+\frac{g}{(h+1)}t_1^{h+1}\right)\right]\left[1-rt_1-gt_1^h\right]dt_1\right\}$$

$$= C_{r2}\,a\left[\frac{T_1^2}{2}+\frac{b}{3}T_1^3+\frac{g}{(h+2)}T_1^{h+2}-\frac{r}{6}T_1^3-\frac{br}{8}T_1^4-\frac{gT_1^{h+2}}{(h+1)(h+2)}-\frac{bg}{(h+1)(h+3)}T_1^{h+3}\right]$$

$$= C_{r2}\,a\left[\frac{T_1^2}{2}+\frac{b}{3}T_1^3-\frac{r}{6}T_1^3-\frac{br}{8}T_1^4+\frac{gh}{(h+1)(h+2)}T_1^{h+2}-\frac{bg}{(h+1)(h+3)}T_1^{h+3}\right] \qquad \ldots(14)$$

(7) Replenishment occurs at t=0. The item cost therefore includes loses due to deterioration as well as the cost of the item sold. The RW present worth cost is

$$IT_r = CI_r = C\left(a\int_0^{T_1} e^{bu+gu^h}\, du \right) = Ca\left[\int_0^{T_1} (1+bu+gu^h)\, du \right]$$

$$= Ca\left[u + \frac{bu^2}{2} + \frac{gu^{h+1}}{h+1} \right]_0^{T_1} = Ca\left[T_1 + \frac{b}{2}T_1^2 + \frac{gT_1^{h+1}}{h+1} \right] \qquad \ldots(15)$$

The present value of the total relevant cost during the cycle is

$$T\cup C(T_1+T_2+T_3) = \frac{1}{T}\Bigg\{ C_1 + C_0 W\left(T_1 - \frac{\alpha}{\beta+1}T_1^{\beta+1} - \frac{rT_1^2}{2} \right)$$

$$+C_{02}\Bigg[W(1-\alpha T_1^{\beta} - rT_1)T_2 + (-W - a + arT_1)\frac{T_2^2}{2} + \left(ar - \frac{ab}{2} + \frac{ab\alpha}{2} \right)\frac{T_2^3}{3}$$

$$+\frac{abr}{8}T_2^4 - \frac{W\alpha T_2^{\beta+1}}{(\beta+1)} + \frac{a\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{ab\alpha T_2^{\beta+3}}{2(\beta+3)} \Bigg] + C_3 Ba$$

$$\left[(1-rT_1-rT_2)\frac{T_3^2}{2} - \frac{rT_3^3}{3} \right] + C_4(1-B)a\Bigg[T_3 - (rT_1+rT_2)T_3 - \frac{1}{2}$$

$$(r+brT_1+brT_2)T_3^2 - \frac{brT_3^3}{3} \Bigg] + C\left[W + BaT_3(1-r(T_1+T_2+T_3)) - brT_3(T_1+T_2+T_3) + bT_3 \right]$$

$$+C_{r2}a\left[\frac{T_1^2}{2} + \frac{bT_1^3}{3} - \frac{rT_1^3}{6} - \frac{br_1^4}{8} + \frac{ghT_1^{h+1}}{(h+1)(h+2)} - \frac{bgT_1^{h+3}}{(h+1)(h+2)} \right]$$

$$+Ca\left(T_1 + \frac{bT_1^2}{2} + \frac{gT_1^{h+1}}{h+1} \right) \Bigg\} \qquad \ldots(16)$$

The optimization problem can be formulated as:
Minimize: $T\,U\,C(T_1, T_2, T_3)$
Subject to: $T_1 \geq 0$, $T_2 \geq 0$, $T_3 \geq 0$

When the system has only RW inventory system

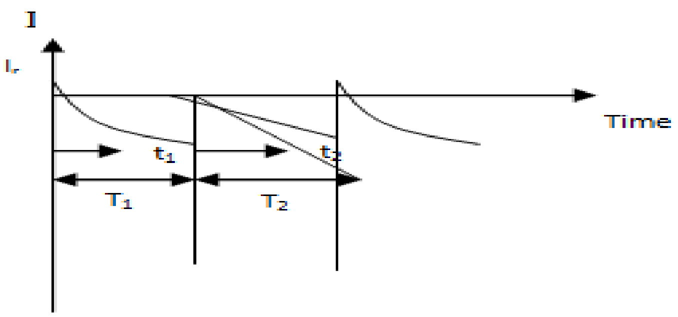

Fig 3. Graphical representation for the RW inventory system(one warehouse-rented)

Fig. 3, shows the graphical representation of the RW inventory system when there is only rented warehouse. The RW inventory level function during T_1 and T_2 time periods are similar to equation (8) and (6). The ordering cost and holding cost of the system are similar to equation (10) and (15).
Shortages occur during T_1 time period. The RW present worth shortage cost is

$$SH = C_3 \left\{ \int_0^{T_2} (Bae^{bt_2} t_2)\, e^{-r(T_1+t_2)} dt_2 \right\}$$

$$= C_3 Ba \left\{ \int_0^{T_2} \left[(1 - rT_1) t_2 - rt_2^3 + bt_2^2 - br\, t_2^2\, T_1 - br\, t_2^3 \right] dt_2 \right\}$$

$$= C_3 Ba \left[(1 - rT_1) \frac{T_2^2}{2} + \left(\frac{-r}{3} + \frac{b}{3} - \frac{br}{3} T_1 \right) T_2^3 - \frac{br}{4} T_2^4 \right] \qquad \ldots(17)$$

Lost sales occur during T_2 time period. The RW present worth lost sales cost is

$$LS = C_4 \left\{ \int_0^{T_2} (1-B)\, ae^{bt_2}\, e^{-r(T_1+t_2)}\, dt_2 \right\} = C_4 (1-B)\, a \left\{ \int_0^{T_2} e^{bt_2}\, e^{-r(T_1+t_2)}\, dt_2 \right\}$$

$$= C_4 (1-B)\, a \left(T_2 - rT_1T_2 - \frac{r}{2} T_2^2 + \frac{b}{2} T_2^2 - \frac{br\, T_1 T_2^2}{2} - \frac{br\, T_2^3}{3} \right) \qquad \ldots(18)$$

Replenishment occurs at t=0 and $t=T_1+T_2=T$. The item cost therefore includes loss due to deterioration as well as the cost of the item sold. The RW present worth item cost is.

$$IT_r = C\left(I_r + Bae^{bt} T_2\, e^{-r(T_1+T_2)} \right) = C\left(a \int_0^{T_1} e^{bu+gu^h} du + Ba\, e^{bt}\, T_2 \sum_{m=0}^{\infty} \frac{(-r(T_1+T_2))^m}{m!} \right)$$

$$= C\,a \left[\int_0^{T_1} e^{bu+gu^h}\, du + B\, e^{bt}\, T_2\, (-r(T_1+T_2)) \right]$$

$$= C\,a \left[\left(T_1 + \frac{bT_1^2}{2} + \frac{g\, T_1^{h+1}}{h+1} \right) + Be^{bt}\, T_2 (1 - r(T_1+T_2)) \right] \qquad \ldots(19)$$

The order quantity in RW per order is

$$Q_r = I_r + aBe^{bt}\, T_2 = a\left(T_1 + \frac{bT_1^2}{2} + \frac{g\, T_1^{h+1}}{h+1} \right) + Be^{bt}\, T_2 = a\left[\left(T_1 + \frac{bT_1^2}{2} + \frac{g\, T_1^{h+1}}{h+1} \right) + Be^{bt}\, T_2 \right]$$

….(20)

Noting that $T=T_1+T_2$, the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, shortage cost, lost sale cost and item cost.

$$T\cup C_r\,(T_1+T_2)=\frac{1}{T}\left\{ C_1+C_{r2}\,a\left[\frac{T_1^2}{2}+\frac{b}{3}T_1^3-\frac{r}{6}T_1^3-\frac{br}{8}T_1^4+\frac{ghT_1^{h+1}}{(h+1)(h+2)}-\frac{bgT_1^{h+3}}{(h+1)(h+3)}\right]\right.$$

$$+C_3\,Ba\left[(1-rT_1)\frac{T_2^2}{2}+\left(\frac{-r}{3}+\frac{b}{3}-\frac{brT_1}{3}\right)T_2^3-\frac{br}{4}T_2^4\right]$$

$$+C_4(1-B)\,a\left(T_2-r\,T_1T_2-\frac{rT_2^2}{2}+\frac{b}{2}T_2^2-\frac{brT_1T_2^2}{2}-\frac{br\,T_2^3}{3}\right)$$

$$\left.+Ca\left[\left(T_1+\frac{bT_1^2}{2}+\frac{gT_1^{h+1}}{(h+1)}\right)+BT_2(1+bt-r\,(T_1+T_2)-br\,t\,(T_1+T_2))\right]\right\}$$

…(21)

When the system has only OW inventory system

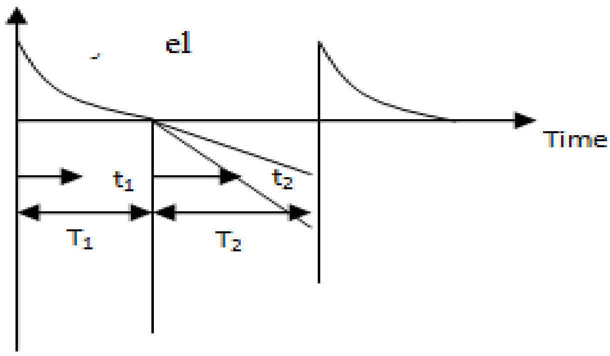

Fig 4. Graphical representation for the OW inventory system(one warehouse-own)

Fig. 4 shows the graphical representation of the OW inventory system when there is only own warehouse. The OW inventory level function during T_1 and T_2 time periods are similar to (8) and (6). The ordering cost, holding cost, shortage cost, and lost sale cost of the system are similar to (10), (15), (18) and (19).

The order quantity in OW per order is

$$Q_0=W+Ba\,e^{bt}\,T_2 \qquad \text{…(22)}$$

Replenishment occurs at t=0 and $t=T_1+T_2$. The item cost therefore includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

$$IT_r=C\left(W+Bae^{bt}T_2\,e^{-r(T_1+T_2)}\right)=C\left[W+Ba\,T_2\left(\sum_{m=0}^{\infty}\frac{(bt)^m}{m!}\right)\left(\sum_{m=0}^{\infty}\frac{(-r\,(T_1+T_2)^m}{m!}\right)\right]$$

$$=C\left[W+Ba\,T_2\,(1+bt-r\,(T_1+T_2)-br\,t\,(T_1+T_2))\right] \qquad \text{…(23)}$$

Noting that $T=T_1+T_2$, the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, lost sale cost and item cost.

$$T\cup C_0\,(T_1,T_2)=\frac{1}{T}\left\{ C_1+C_{02}\,a\left[\frac{T_1^2}{2}+\frac{b}{3}T_1^3-\frac{r}{6}T_1^3-\frac{br}{8}T_1^4+\frac{\alpha\beta\,T_1^{\beta+1}}{(\beta+1)(\beta+2)}-\frac{b\,\alpha\,T_1^{\beta+3}}{(\beta+1)(\beta+3)}\right]\right.$$

$$+C_3\,Ba\left[\frac{(1-rT_1)\,T_2^2}{2}+\left(\frac{-r}{3}+\frac{b}{3}-\frac{brT_1}{3}\right)T_2^3-\frac{br}{4}T_2^4\right]$$

$$+C_4(1-B)\,a\left(T_2-r\,T_1T_2-\frac{rT_2^2}{2}+\frac{bT_2^2}{2}-\frac{brT_1T_2^2}{2}-\frac{br\,T_2^3}{3}\right)$$

$$\left.+C\left[W+Ba\,T_2(1+bt-r\,(T_1+T_2)-br\,t\,(T_1+T_2))\right]\right\} \qquad \ldots(24)$$

Solution Procedure:

To derive the optimal solutions, the following classical optimization technique is used. Step-1: Take the partial derivatives of TUC(T_1, T_2, T_3) with respect to T_1, T_2 and T_3, and equating the results to zero. The necessary conditions for optimality are

$$\frac{\partial\, T\cup C(T_1,T_2,T_3)}{\partial\, T_1}=0,\ \frac{\partial\, T\cup C(T_1,T_2,T_3)}{\partial\, T_2}=0$$

And $$\frac{\partial\, T\cup C(T_1,T_2,T_3)}{\partial\, T_3}=0$$

Step-2: The simultaneous equations above can be solved for T_1^*, T_2^* and T_3^*.

Step-3: With T_1^*, T_2^* and T_3^* found in step 2, derive. $T\cup C^*(T_1^*, T_2^*, T_3^*)$

Step-4: With T_1^*, T_2^* and T_3^* found in step 2, derive the order quantity in OW, which is W+BdT_3 and the RW order quantity, which is

$$a\left(T_1+\frac{b}{2}T_1^2+\frac{g}{h+1}T_1^{h+1}\right).$$

III. CONCLUSION

In this paper, an inventory model is developed to determine the optimal replenishment cycle for two-warehouse inventory problem under inflation, varying rate of deterioration and partial backordering. The model assumes limited warehouse capacity of the distributors. Classical optimization technique is used to derive the optimal replenishment policy. When there is only rented or own warehouse in the inventory system, the total presented values of the total relevant cost per unit time are higher than the two-warehouse model. We can see that the deterioration rate, the inflation and the backordering rate affect the total cost of the system. In order to optimize the system, the decision maker must develop the most economical replenishment strategy.

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