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Integral Solutions of Non-Homogeneous Biquadratic Diophantine Equation $x^4 - y^4 = 34\ (z + w)\ p^2$ With Five Unknowns

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Abstract: *The non-homogeneous biquadratic Diophantine equation given by $x^4 - y^4 = 34\ (z +$ $w\ p2$ is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.*

Keywords: *Non-homogeneous Equation, Integral Solutions, Polygonal Numbers, Pyramidal Numbers and Special Numbers*

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I. Introduction

Biquadratic Diophantine equations homogenous and non–homogenous are rich in variety [1-3]. For an extensive review of sizable literature and various problems, one may refer [4-18]. In this communication, we consider yet another interesting biquadratic equation $x^4 - y^4 = 34\ (z + w)\ p^2$ and obtain infinitely many non-trivial integral solutions. A few interesting relations between the solutions and special polygonal numbers are presented.

Notations Used

- $t_{m,n}$- Polygonal number of rank ‘n’ with size ‘m’
- CP_n^6 - Centered hexagonal Pyramidal number of rank n
- Gno_n – Gnomic number of rank ‘n’
- FN_A^4– Figurative number of rank ‘n’ with size ‘m’
- Pr_n - Pronic number of rank ‘n’
- P_n^m- Pyramidal number of rank ‘n’ with size ‘m’
- $carl_n$ -- Carol number
- ky_n - Keynea number
- Tha_n - Thabit number
- S_n – Star number of rank n
- SO_n - Stella octagonal number of rank n

II. Method of Analysis

The Diophantine equation representing the biquadratic equation with five unknowns to be solved for its non zero distinct integral solutions is

$$x^4 - 4 = 34(z + w)p^2 \qquad (1)$$

Consider the transformation

$$\left.\begin{aligned} x &= u + v \\ y &= u - v \\ z &= 2uv + 1 \\ p &= 2uv - 1 \end{aligned}\right\} \qquad (2)$$

On substituting (2) in (1) , we get

$$u^2 + v^2 = 17\ p^2 \qquad (3)$$

Now,We illustrate methods of obtaining non Zero distinct integer solutions to (1)

Pattern I

Assume

$$p = A^2 + B^2 = (A + iB)(A - iB) \qquad (4)$$

Let $u^2 + v^2 = 16\, p^2 + p^2$

$$\frac{u + p}{4p + v} = \frac{4p - v}{u - p}$$

Employing the method of factorization, we get the system of equations

$$\left.\begin{aligned} u(A,B) &= A^2 - B^2 + 8AB \\ v(A,B) &= -4A^2 + 4B^2 + 2AB \\ p(A,B) &= A^2 + B^2 \end{aligned}\right\}$$

Substituting the values of u,v and p in (2),the nonzero distinct integral values of x,y,z ,w and p are given by

$$\left.\begin{aligned} x = x(A,B) &= -3A^2 + 3B^2 + 10AB \\ y = y(A,B) &= 5A^2 - 5B^2 + 6AB \\ z = z(A,B) &= -8A^4 - 8B^4 + 48A^2B^2 - 60A^3B + 60AB^3 + 1 \\ w = w(A,B) &= -8A^4 - 8B^4 + 48A^2B^2 - 60A^3B + 60AB^3 - 1 \\ p = p(A,B) &= A^2 + B^2 \end{aligned}\right\}$$

Properties

1. $y(A,A) + x(A,A) - 2p(A,A) - 16\, t_{4,b} \equiv 0$
2. $y(1,1) + x(1,1) - 2p(1,1)$ is a perfect square
3. $5x(A,1) + 3y(A,1) \equiv 0 (\mathrm{mod}\ 68)$
4. $z(A,B) - w(A,B)$ is a Thabit number
5. $z(1,1) + w(1,1)$ is a cubic integer
6. $-6[z(1,0) + w(1,0)]$ is a nasty number
7. $31[y(1,1) + p(1,1)]$ is a perfect number
8. $z(A,1) + w(A,1) + 192Fn_A^4 - 80Pr_A + 120CP_A^6 - 16Gno_A \equiv 0 (\mathrm{mod}\ 8)$

Pattern II

Equation (3) as

$$u^2 + v^2 = 17p^2$$

Where $17 = (4 + i)(4 - i)$ (5)

Using (4)and (5) in (3) and writing (3) in factorization form as

$(u + iv)(u - iv) = (4 + i)(4 - i)[(A + iB)(A - iB)]^2$

which is equivalent to the system of equation

$u + iv = (4A^2 - 4B^2 - 2AB) + i(A^2 - B^2 + 8AB)$

Equating real and imaginary parts ,we get

$u = 4A^2 - 4B^2 - 2AB$

$v = A^2 - B^2 + 8AB$

On substituting the values of u and v in (2) the nonzero distinct integral values of x, y, z, w and p satisfying (1) are given by

$$\left.\begin{aligned} x = x(A,B) &= 5A^2 - 5B^2 + 6AB \\ y = y(A,B) &= 3A^2 - 3B^3 - 10AB \\ z = z(A,B) &= 8A^4 + 8B^4 - 48A^2B^2 + 60A^3B - 60AB^3 + 1 \\ w = w(A,B) &= 8A^4 + 8B^4 - 48A^2B^2 + 60A^3B - 60AB^3 - 1 \\ p(A,B) &= A^2 + B^2 \end{aligned}\right\}$$

Properties

1. $x(A,A) + y(A,A) + 8p(A,A) - 16Pr_A \equiv 0 (\mathrm{mod}\ 20)$
2. $x(1,B) + y(1,B) + 8p(1,B) \equiv 16 (\mathrm{mod}\ 4)$
3. $z(A,1) + w(A,1) - 192FN_A^4 - 60So_A + 60Pr_A + 20t_{4,A} - 16 \equiv 0$
4. $z(1,B) + w(1,B) - 16Bi_B - 16So_B + 16S_B + 18Gno_A - 14 \equiv 0$
5. $z(1,1) + w(1,1)$ is a cubic integer
6. $z(1,0) + w(1,0)$ is a perfect number

Pattern III

Equation (3) as

$$u^2 + v^2 = 17p^2$$

Where $17 = (1+4i)(1-4i)$ (6)

Using (4)and (6) in (3) and writing (3) in factorization form as

$(u + iv)(u - iv) = (1 + 4i)(1 - 4i)[(A + iB)(A - iB)]^2$

which is equivalent to the system of equation

$u + iv = (A^2 - B^2 - 8AB) + i(4A^2 - 4B^2 + 2AB)$

Equating real and imaginary parts ,we get

$u = A^2 - B^2 - 8AB$

$v = 4A^2 - 4B^2 + 8AB$

On substituting the values of u and v in (2)the nonzero distinct integral values of x, y, z, w and p satisfying (1) are given by

$$\left.\begin{array}{c} x = x(A,B) = 5A^2 - 5B^2 - 6AB \\ y = y(A,B) = -3A^2 - 3B^3 - 10AB \\ z = z(A,B) = 8A^4 + 8B^4 - 48A^2B^2 - 60A^3B + 60AB^3 + 1 \\ w = w(A,B) = 8A^4 + 8B^4 - 48A^2B^2 - 60A^3B + 60AB^3 - 1 \\ p = p(A,B) = A^2 + B^2 \end{array}\right\}$$

Properties

1. $x(3,2) + y(3,2) + 2p(3,2)$ is a nasty number
2. $z(A,1) + w(A,1) - 192FN_A^4 + 120CP_A^6 + 30Oct_A - 30Gno_A - 46 \equiv 0$
3. $y(1,-1) + 3p(1,-1)$ is a perfect square
4. $3x(A,1) + 5y(A,1) + 68A \equiv 0$
5. $x(B,B) + p(B,B) + 2\,t_{4,\beta} \equiv 0$

Pattern IV

Equation (3) as

$u^2 + v^2 = 17p^2 * 1$ (7)

Write 1 as

$1 = \frac{(A^2-B^2+2iAB)(A^2-B^2-2iAB)}{(A^2+B^2)^2}$ (8)

Using (4)and (8) in (3) and writing (3) in factorization form as

$(u + iv)(u - iv) = (1 + 4i)(1 - 4i)[(A + iB)(A - iB)]^2 \frac{(A^2-B^2+2iAB)(A^2-B^2-2iAB)}{(A^2+B^2)^2}$

which is equivalent to the system of equation

$u + iv = (1 + 4i)(A + iB)^2 \frac{(A^2-B^2-2iAB)}{(A^2+B^2)}$

Equating real and imaginary parts ,we get

$u = \frac{1}{(A^2+B^2)}(A^4 + B^4 - 6A^2B^2 - 16A^3B + 16AB^3)$

$v = \frac{1}{(A^2+B^2)}(A^4 + 4B^4 - 24A^2B^2 + 4A^3B - 4AB^3)$

On substituting the values of u and v in (2)the nonzero distinct integral values of x, y,z,w and p satisfying (1) are given by

$$\left.\begin{array}{c} x = x(A,B) = \frac{1}{A^2+B^2}(5A^4 + 5B^4 - 30A^2\,B^2 - 12A^3B + 12AB^3) \\ y = y(A,B) = \frac{1}{A^2+B^2}(-3A^4 - 3B^4 + 18A^2\,B^2 - 20A^3B + 20AB^3) \\ z = z(A,B) = \frac{2}{(A^2+B^2)^2}(4A^8 + 4B^8 - 60A^7B - 112A^6B^2 + 420A^5B^3 + 280A^4B^4 - 420A^3B^5 - 112A^2B^6 + 60AB^7) + 1 \\ w = w(A,B) = \frac{2}{(A^2+B^2)^2}(4A^8 + 4B^8 - 60A^7B - 112A^6B^2 + 420A^5B^3 + 280A^4B^4 - 420A^3B^5 - 112A^2B^6 + 60AB^7) - 1 \\ p = p(A,B) = A^2 + B^2 \end{array}\right\}$$

Properties

1. $x(A,0) - 5t_{4,A} \equiv 0$
2. $z(1,1) + w(1,1) + x(1,1)$ is a nasty number
3. $z(0,B) - 8Bi_A - 1 \equiv 0$
4. $y(1,-1) + 3p(1,-1)$ is a perfect square
5. $p(2^n, 2^n) = Carl_n + Ky_n + 2$
6. $2p(A,A)$ is a nasty number

III. Conclusion

In this paper, we have presented four different patterns of non- zero distinct integer solutions of biquadratic Diophantine equation $x^4 - y^4 = 34\ (z + w)\ p^2$ and relations between solutions and special numbers are also obtained. To conclude, one may search for other patterns of solutions and their corresponding properties.

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