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# Viscous Dissipation and Radiation Effects on MHD Convective Flow Past A Vertical Porous Plate with Injection

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***Abstract:*** *The boundary layer steady flow and heat transfer towards a porous platein presence of a magnetic field and injection is presented in this analysis.Viscous dissipation and thermal radiationtermsare incorporated in thetemperature equation. Similarity transformations are used to convert the partial differential equationscorresponding to the momentum and energy equations into non-linear ordinary differentialequations. Numerical solutions of these equations are obtained by shooting technique. An increase in the injection parameter leads to a decrease in the fluid velocity.The greater viscous dissipative heat causes an increase in the fluid temperature.*

***Keywords****: Viscous dissipation; Porous plate; MHD; Thermal radiation; Similarity Transformation; Injection.*

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## I. Introduction

The study of fluid flow and heat transfer over a porous medium is a subject of interest for many researchers working in the area of the two-dimensional flows. The reasons lie in the fact that such kinds of investigations have applications in the manufacturing industry. Further, the flow in a porous media is used to study the migration of underground water, movement of oil, gas and water through the reservoir, water purification and ceramic engineering, the aerodynamic extrusion of plastic sheets, blood flow problems, the cooling of an infinite metallic plate in a cooling bath, textile and paper industries etc.The initial work was investigated by Sakiadis [1] for two-dimensional boundary layer flow when the plate is moving with constant velocity. The Sakiadis's problem for heat transfer analysis was studied by Erickson et al. [2].

Kumaran et al. [3] analyzed the MHD boundary layer flow of an electrically conducting fluid over a stretching permeable surface with suction/ injection. Hassanien and Al Arabi [4] presented the problem of boundary layer unsteady mixed convection flow near the stagnation point on a heated vertical plate through a porous medium. Hayat et al. [5] investigated MHD stagnation point flow and heat transfer through a porous space bounded by a permeable surfaceThermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators, power generation systems are some important applicationsofradiative heat transfer from a vertical wall to conductive gray fluids. Gupta and Gupta [6] analyzed the momentum, heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing. SwathiMukhopadhyay [7] studied the slip effects on MHD boundary layer flow over exponential stretching sheet with thermal radiation.Subhas Abel et al. [8] studied the effect of viscous and Joules dissipation on MHD flow over a porous nonlinear vertical stretching sheet with partial slip. Kumar [9] investigated the study of radiation and viscous dissipation effects over a stretching surface subjected to variable heat flux in presence of transverse magnetic field.Recently, MadhusudhanaRaoet al [10] studied the steady MHD fluid flow past a vertical porous plate with injection and thermal radiation effects. Kishan et.al [11] studied the two-dimensional steady nonlinear MHD boundary layer flow of an incompressible, viscous, electrically conductive and Boussinesq fluid flowing over a vertical stretching surface in the presence of uniform magnetic field by taking into account the viscous dissipation with heat, mass transfer chemical reaction and thermal stratification effects. Kairi et al. [12] investigated the combined effect of viscous dissipation and radiation on natural convection in non-Darcy porous medium saturated with non-Newtonian fluid of variable viscosity. El-Arabawy [13] studied the effects of suction/injection and chemical reaction on mass transfer over a stretching surface.Reddy et al. [14] considered heat transfer in hydro magnetic rotating flow of viscous fluid through non-homogeneous porous medium with constant heat source/sink. Ravikumar et al. [15], investigated heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source.

Umamaheswar et al. [16] considered combined radiation and Ohmic heating effects on MHD free convective visco-elastic fluid flow past a porous plate with viscous dissipation.Rao et al. [17] addressed an unsteady MHD mixed convection of a viscous double diffusive fluid over a vertical plate in porous medium with chemical reaction, thermal radiation and Joule heating. Umamaheswar et al. [18] reported an unsteady MHD free convective visco-elastic fluid flow bounded by an infinite inclined porous plate in the presence of heat source, viscous dissipation and Ohmic heating. Raju et al. [19] investigated MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule's heating. Raju et al. [20] examined heat transfer effects on a viscous dissipative fluid flow past a vertical plate in the presence of induced magnetic field. Vidyasagar et al. [21]considered an unsteady MHD free convection flow of a viscous dissipativefluid past an infinite vertical porous plate in the presence of radiation, thermal diffusion and chemical reaction. Praveena et al. [22] addressed an unsteady hydromagnetic free convective heat transfer flow of visco-elastic fluid through porous medium with heat source and viscous dissipation. Ananda Reddy et al. [23] considered thermal and solutal buoyancy effects on viscous dissipative and chemically reactive fluid flow past a uniformly moving plate with variable suction. Umamaheswar et al. [24] investigated an unsteady MHD free convective double diffusive visco-elastic fluid flow past an inclined permeable plate in the presence of viscous dissipation and heat absorption.

Due to the importance of injection and the presence of viscous dissipation, the boundary layer steady flow and heat transfer towards a porous platein presence of a magnetic field is presented.The partial differential equations governing the flow have been transferred into a system of ordinary differential equations using similarity transformations and solved numerically using Runge-Kutta method with shooting technique. Numerical calculations are taken up to desire level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically.

## II. Mathematical Formulation

Consider a two dimensional incompressible viscous electrically conductingfluid past a porous plate with injection.In rectangular Cartesian coordinate system, we take x-axis along the plate in the direction of flow and y-axis normal to it. The surface of the plate is maintained at a uniform temperature $T_w$.The flow is confined to y> 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. Governing equations describing the conservation of mass, momentum and energy equations are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\gamma}{k'}u-\frac{\sigma B^2}{\rho}u \; (2) \; u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)+\frac{\upsilon}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

with the boundary conditions;

$$\begin{aligned} &u=l\,,\quad v=v_w\,,\quad T=T_w \quad at \quad y=0 \\ &u\to 0,\; T\to T_\infty \qquad as\; y\to\infty \end{aligned} \quad (4)$$

Where $\upsilon$ is kinematic viscosity and we consider the following dimensionless similarity transformations

$$\eta=y\sqrt{\frac{l}{\gamma x}}\,,\quad \psi=\sqrt{l\gamma x}\,f(\eta)\,,\quad T=T_\infty+\theta(T_w-T_\infty)$$

$$M=\frac{2x\sigma B^2}{b\rho},\, K=\frac{2\gamma x}{K'b^2},\, \Pr=\frac{\gamma\rho C_p}{k}\,,\quad Q=\frac{\gamma x Q_0}{k\,b(T_w-T_\infty)}\,, \qquad (5)$$

$$F_w=-2v_1\sqrt{\frac{x}{b\gamma}}\,,\quad E_c=\frac{\gamma l^2\rho}{(T_w-T_\infty)\,k}$$

Where $f(\eta)$ is dimensionless stream function. In the view of equation (5), the equations (2) and (3) are reduced to the following ordinary nonlinear differential equations

$$2f'''+f\,f''-(M+K)f'=0 \; (6)$$

$$2\theta'' + P_r f\,\theta' + Q\theta + 2E_c\left(f''\right)^2 = 0 \quad (7)$$

and the corresponding boundary conditions in non-dimensional quantities are given by,

$$\begin{aligned} &f' = 1\,,\ f = F_w\,,\ \theta = 1 \quad at \quad \eta = 0 \\ &f' \to 0\,,\ \theta \to 0 \qquad\qquad as \quad \eta \to \infty \end{aligned} \qquad (8)$$

## III. Solution Of The Problem

The set of non linear ordinary differential equations (6) and (7) together with the boundary conditions (8) is solved by fourth-order Runge-Kutta method along with Shooting technique for the prescribed parameters Magnetic parameter (M), Permeability of porous medium (K), Eckert number (Ec), Heat source parameter (Q), Prandtl number (Pr), Radiation parameter (R) and Injection parameter ($F_w$). In the boundary conditions (8), there are four asymptotic boundary conditions and hence there are four unknown surface conditions $f''(0)$, $f'(0)$, $\theta'(0)$ . Values of these unknown surface conditions are obtained by shooting technique. A program is set up for the above mentioned procedure along with the Runge-kutta method to solve the equations (6) and (7) with the boundary conditions (8).

## IV. Results And Discussion

The dimensional less velocity and temperature profiles for different value of Magnetic parameter M with K=1, Pr=0.71, Q=0.3, Fw=1, Ec=0.5are shown in Fig.1 and Fig.2. As M increases, velocity found to decrease (Fig.1) and the temperature increases (Fig.2). This is because with the increase in M, Lorentz force increases and it produces more resistance to the fluid flow. In addition with this the thermal boundary layer thickness also increases as M increases.

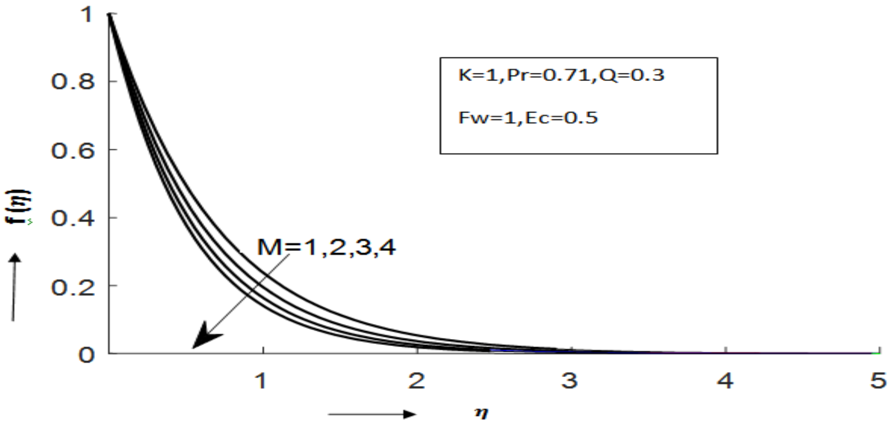

**Fig.1.** Effects ofMagnetic parameter (M) on Velocity

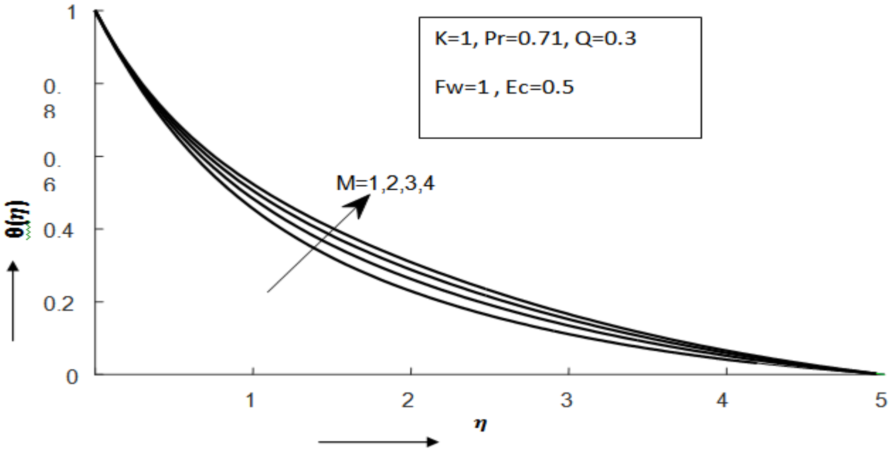

**Fig.2.** Effects ofMagnetic parameter (M) on temperature

The influence of permeability of porous medium K with M=1, Pr=0.71, Q=0.3, $F_w$=1, Ec=0.5on velocity and temperature profiles is represented in Fig.3 and Fig.4 respectively. As K increases velocity decreases (Fig.3) and temperature increases (Fig.4). In other words this increase of permeability of porous medium decreases the thickness of momentum boundary layer which eventually increases the heat transfer.

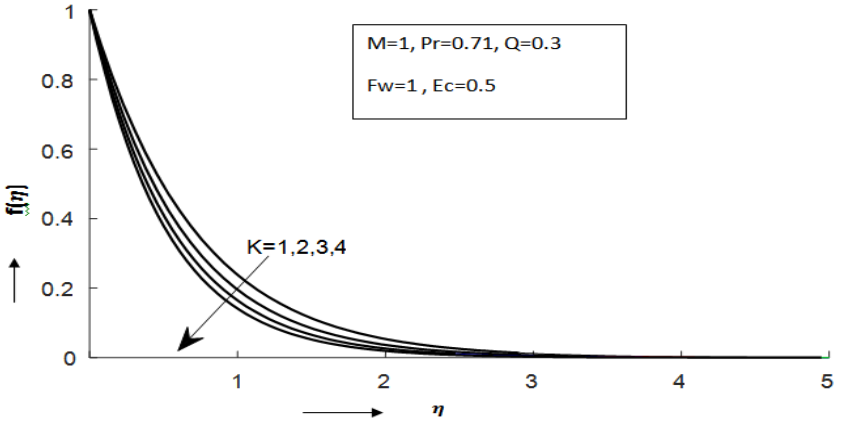

Fig.3. Effects of Permeability parameter(K) on Velocity

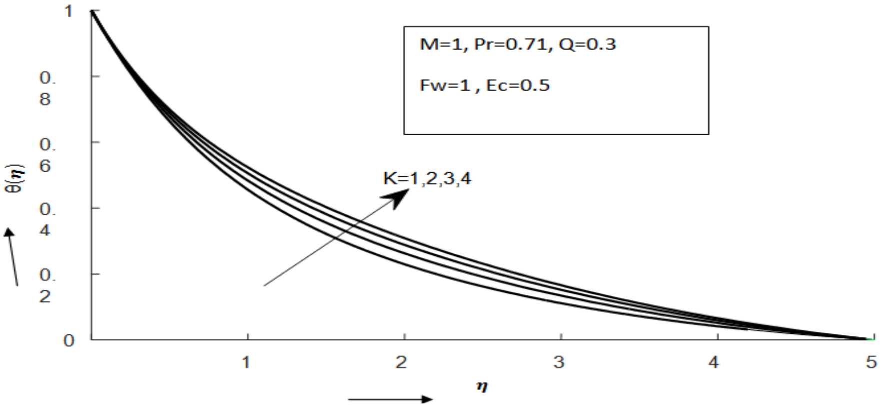

**Fig.4.** Effects of Permeability parameter (K) on Temperature

The effects of Radiation parameter Qwith K=1, Pr=0.71, M=1, $F_w$=1, Ec=0.5on the temperature are represented in Fig.5 and it is observed that heat transfer increases for increasing value of Q. It is noticed from Fig.6 (with K=1, Q=0.3, M=1, $F_A$=1)the temperature decreases as Pr increases, and this is due to verity that thermal boundary layer decreases with an increase in Pr.

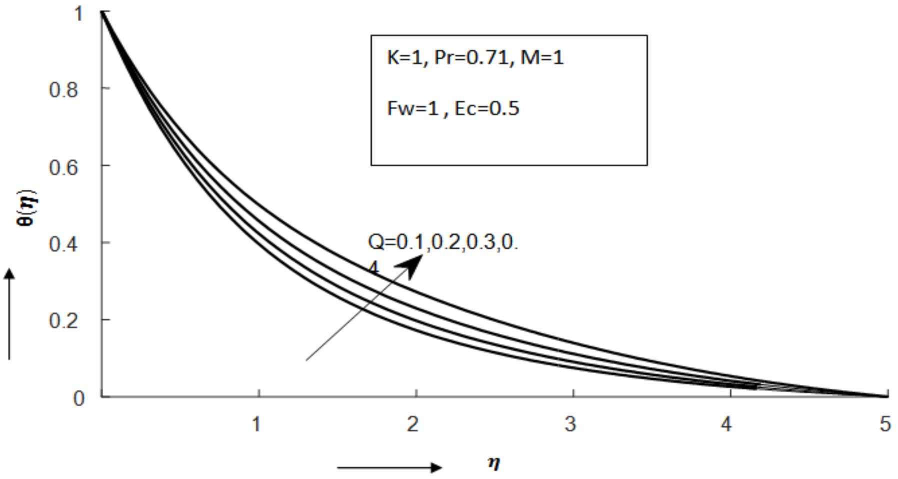

**Fig.5.** Effects of Heat source parameter(Q) on Temperature

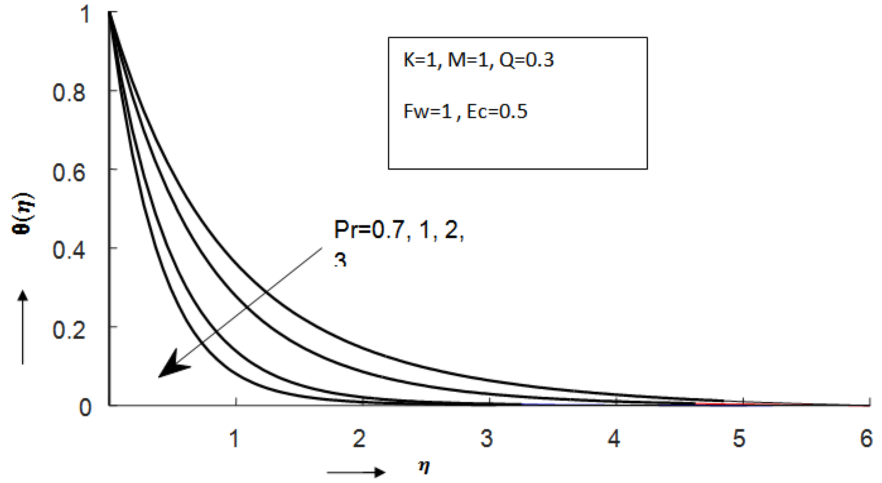

**Fig.6.** Effects of Prandtl number (Pr) on Temperature

The velocity component for different values of injection parameter $F_w$withK=1, Pr=0.71, M=1, Ec-0.5, Q=0.3 is portrayed in Fig.7. It is observed that for increasing value of $F_w$ the velocity within the boundary layer exponentially decreases. Due to wall injection ($F_w>0$), resistance of the fluid increases and has a conduct to condense the velocity of the fluid flow, which is observed from Fig.7.But the wall injection ($F_w<0$) produces the reverse effect. The temperature profiles for the various values of $F_w$with K=1, Pr=0.71, Ec=0.5, M=1, Q=0.3 are shown in Fig.8. Similar to velocity of the fluid wall injection ($F_w>0$) has a propensity to reduce the thermal boundary layer thickness, which can be shown in Fig.8. But when $F_w<0$, thermal boundary layer thickness increases. As of this fact, the temperature within thethermalboundarylayerprogressivelydecreasesas$F_w$increase.

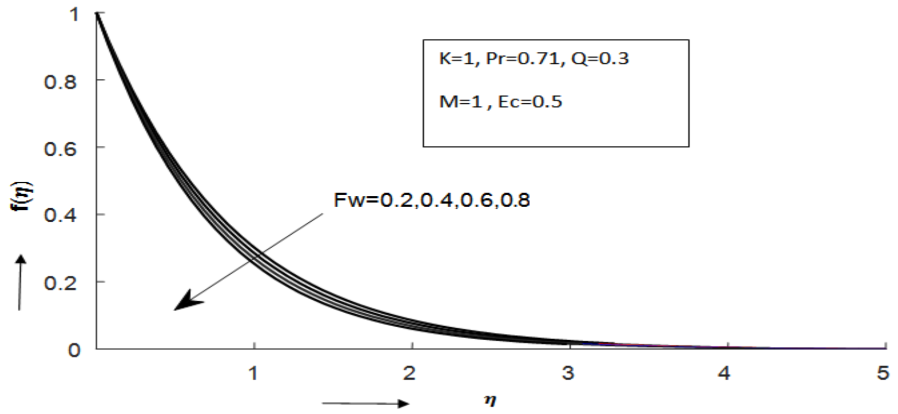

**Fig.7.** Effects of Injection parameter (Fw) on Velocity

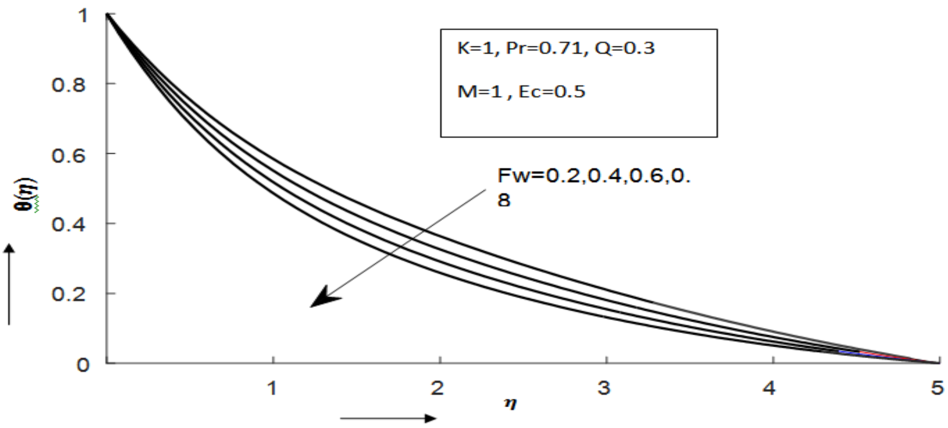

**Fig.8.** Effects of Injection parameter (Fw)on Temperature

The temperature profiles for the various values of Eckert number $E_c$ with K=1, Pr=0.71, Fw=0.2, M=1, Q=0.3 is shown in Figure 9. The renovation of kinetic energy into internal energy by work done in opposition to the viscous fluid stresses is incorporated by the Eckert number. The positive Eckert number implies the cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes an increase in the fluidtemperatureaswellasthefluid velocity.

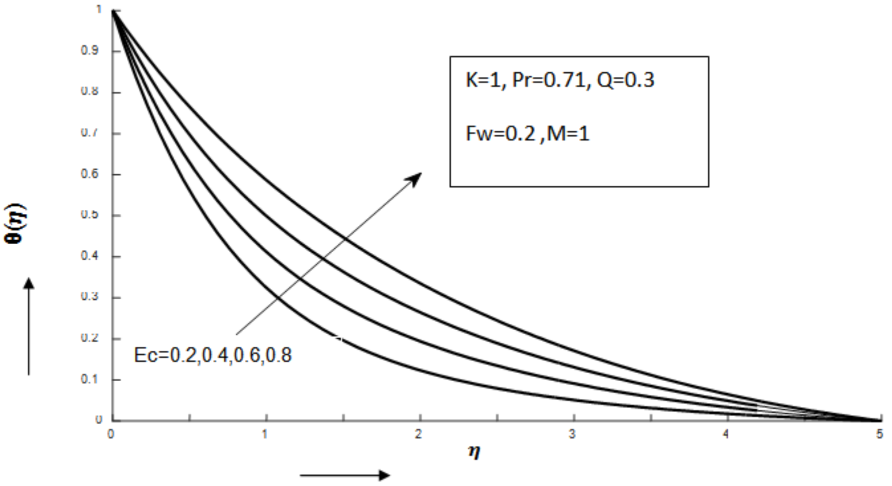

**Fig.9.** Effects of Eckert number (Ec) on Temperature

From the Table-1, it is seen that skin friction coefficient increases with the increasing values of K, M and no effect of Ec and Q is observed on skin friction. The rate of heat transfer coefficient is increases with an increase in M,K and Q but the opposite trend is observed with an increase in Ec.

| M | K | Q | Ec | $-f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|---|
| **1** | 1 | 0.1 | 0.2 | 1.371050 | -0.854592 |
| **2** | 1 | 0.1 | 0.2 | 1.573720 | -0.838500 |
| **3** | 1 | 0.1 | 0.2 | 1.749786 | -0.825602 |
| 1 | **1** | 0.1 | 0.2 | 1.371050 | -0.854592 |
| 1 | **2** | 0.1 | 0.2 | 1.573720 | -0.838500 |
| 1 | **3** | 0.1 | 0.2 | 1.749786 | -0.825602 |
| 1 | 1 | **0.1** | 0.2 | 1.371050 | -0.969692 |
| 1 | 1 | **0.2** | 0.2 | 1.371050 | -0.916343 |
| 1 | 1 | **0.3** | 0.2 | 1.371050 | -0.854592 |
| 1 | 1 | 0.1 | **0.2** | 1.371050 | -0.541120 |
| 1 | 1 | 0.1 | **0.4** | 1.371050 | -0.750101 |
| 1 | 1 | 0.1 | **0.6** | 1.371050 | -0.959083 |

**Table.1**.Numerical values of $-f''(0)$ and $-\theta'(0)$ for various values of the flow parameters with Pr=0.71andFw=1;

## V. Conclusions

In this paper the stagnation flow of MHD fluid in presence of viscous dissipation, porous medium, injection and thermal radiation parameters is studied and based on this investigation the following observations are made. The velocity decreases with the increasing values of Magnetic parameter whereas temperature increases. The temperature increases with an increase in Radiation parameter.An increase in the injection parameter leads to decrease in the fluid velocity. This is due to the fact that the suction stabilizes the boundary layer growth and the temperature within the thermal boundary layer gradually decreases as suction parameter increases.The temperature decreases when Prandtl number increases. This is due to the fact that the fluid with higher Prandtl number has relatively low thermal conductivity. An increase of permeability of porous medium decreases the thickness of momentum boundary layer which eventually increases the heat transfer. Thegreater viscous dissipative heat causes an increase in the fluid temperature.

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