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Multi-Objective Job Shop Scheduling Using a Lexicographic Procedure

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Abstract: *In this paper, amulti-objectivejob shop scheduling model using Goal Programming (GP) lexicographic procedure is formulatedbased onmakespan, total tardiness and total earliness criteria.The objectives of minimizing both the makespan and total earliness aregenerallyconsidered inconsistent or conflicting objectives. Conversely, the objectives of minimizing the makespan and total tardiness are usuallyconsistent objectives. However,the objectives of minimizing total earliness and total tardiness are typicaly two independent objectives.The effect of priority ordering and objective deviation are discussed through thirty3x3 and four 10x10 problems.*

Keywords:*multi-objectives, goal programming; job shop; mixed integer programming; lexicographic procedure.*

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I. Introduction

The scheduling process allocatesavailable resources to the required tasks over a given period of time to optimize one or more objectives, and such processes are regularly used in many business industries. Job-shop scheduling affects the resource utilization of the associated facility. In addition, it affects the cost of just-in-time (JIT) manufacturing,including the total tardiness and total earliness of jobs.Decision makers often deal with problems that involve multiple usually conflicting, criteria [1]. Inlexicographic procedures, objectives are prioritized and ordered according tothe prioritization results. Initially, the optimal value of the first objective is found. Then, the objective function is turned into a constraint, such that its value does not differ from the optimal value by more than a certain pre-defined deviation. The next objective is then optimized, and the process is repeated for all objectives[2].Job shop scheduling problems are branch of production planning, which includes some of the most complex problems in combinatorial optimization. Proper scheduling has always been one of the important success factors for production systems. G. Zhang[3].Jeffrey W Herrmann[4]described JSSP as an optimization problem that can be a set of jobs, each with one or more operations. The operations of a job have to be processed on a particular set ofmachines in a specified sequence.Many approaches using both mathematical formulations and heuristic methods have been developed to solve this problem. Mathematical formulations are used to address the JSSP situation of small size in a reasonable computational time. However, due to unacceptable computation time,exact algorithms such as branch and bound method and mixed integer linear programming method cannot be appliedto the middle and large-scale problems with sufficient time. Li X, Yin Mand Valerie Belton, Mark D Elder[5, 6]proposed heuristic algorithms based on the constructive operationto solve the large-sized scheduling problems.

AbysonScaria et al. [7]mentioned that most of theresearchers in production scheduling are concerned with the optimization of a single criterion. However, the performance of aschedule often involves more than one aspect and, therefore requires a multi-objective treatment. They considered minimization of makespan andtotal tardiness in theirwork.Early studies addressed single machine scheduling and multi-machine scheduling. In 1959, Wagner [8]addressed the flow shop problem to minimize the makespanfor three machines. In addition, Baker and Keller [9]formulated a mixed integer programming (MIP) model for single machine scheduling with a tardiness objective.Manne[10]focused on the job shop problem with the aim of minimizing the makespan.
M. S. Al-Ashhab et al.[11]formulated four different models using MIPto optimize makespan with and without considering the due date, total earliness, total tardiness.HamdyElwany, et. al.[12]performed an elaborative study of production scheduling and various traditional and modern approaches toscheduling problems explaining the need for different scheduling approaches in a manufacturing industry

In this research, a multi-objectivejob shop scheduling model using GPlexicographic procedure is formulated with makespan, total tardiness and total earliness criteria.The objectives of minimizing both the makespan and total earliness are generally considered inconsistent or conflicting objectives. Conversely, the objectives of minimizing the makespan and total tardiness are usuallyconsistent objectives. However, the objectives of minimizing total earliness and total tardiness are typicaly two independent objectives. The effect of priority ordering and objective deviation are discussed through thirty 3x3 and four 10x10 problems.

The following assumptions are considered in the model:

1) All jobs are ready for processing at time zero.
2) All machines are available at time zero.
3) Only one job can be processed by a machine at any instant in time.
4) Recirculation is not allowed.
5) The processing times are known, fixed, and independent of the sequence.
6) The set-up time for any operation is included in the processing time.
7) The transportation time required for the movement of jobs between machines is assumed to be negligible.
8) Pre-emption is not permitted, i.e., once an operation has started on a machine, it must be completed before anotheroperation can be started on that machine.

II. Model Formulation

In this research, the notations used to develop the MIP formulations are:

Parameters:

N: Number of jobs
M: Number of machines
Phj: Processing time for job j on machine h
Dj: Due date of job j
NUMT: No. of machines (tasks) for each job
SEQ: Processing sequence array
NUMJ: No. of jobs per machine J
DISJ: Disjunction array.

Decision Variables:

Cj:Completion time of job j
Shj:Starting time of job j on machine h
Ej:Earliness of job j = (Dj - Cj) if Dj>Cj and 0 otherwise
Tj:Tardiness of job j = (Cj - Dj) if Cj > Dj and 0 otherwise

2.1. Model Formulation

The model considers three objectives: minimizingmakespan(Equation (1)), minimizing total tardiness(Equation (2)), and maximizing total earliness(Equation (3)).

$$\text{minimize f1} = \text{Cj}, \forall j \in N \tag{1}$$

$$\text{minimize f2} = \sum_{j \in N} \text{Tj} \tag{2}$$

$$\text{minimize f3} = \sum_{j \in N} \text{Ej} \tag{3}$$

Subject to.

$$\left(S_{hi} - S_{hj}\right) \geq P_{hj} - MY_{hij}, \forall i, j \in N, \forall h \in M \tag{4}$$

$$\left(S_{hi} - S_{hj}\right) \geq P_{hj} - M(1 - Y_{hij}), \forall i, j \in N, \forall h \in M \tag{5}$$

$$\sum_{h \in M} \left(S_{SEQ(j,l),j} + P_{SEQ(j,l),j}\right) \geq \sum_{h \in M} S_{SEQ(j,l+1),j}, \forall j \in N, \forall l \in M - 1 \tag{6}$$

Constraints (4) and (5) are two mutually exclusive constraints. One of constraints must be relaxed when job*i* precedes job *j*or *j* precedes *i* on machine *k* to avoid overlapping between tasks (disjunction constraints).

a) Conjunctive (Precedence) Constraints

Constraint (6) is called the conjunction constraint which ensures that the processing sequence or operational precedence between the tasks is satisfied.The model is coded in MOSEL language, solved using Xpress-MP 7.9 software and ran on an Intel® Core™ i3-2310M CPU @2.10 GHz (3 GB of RAM).

III. Computational ResultsAnd Analysis

In the following sections, the effect of objectives' priority ordering is presented and analysed for six3x3 problems assuming zero allowed deviation. The effects ofboth priority ordering and objective deviation are discussedforthirty3x3 problems.Four 10x10 problems are solved and analysed to prove the ability of the model for solving larger problems.

3.1 Effect Of Priority Ordering

In this section, the effects of objectivepriority ordering are analysedand assesed through the results of six 3x3 problems assuming zero allowable deviation.Table 1shows the assumed priority orders of the three objectives in the above mentioned six problems.

Table 1. The priority orders of the three objectives of the six problems.

Exp. No.	Makespan	Total Tardiness	Total Earliness
1	**1**	**2**	**3**
2	**1**	**3**	**2**
3	**2**	**1**	**3**
4	**3**	**1**	**2**
5	**2**	**3**	**1**
6	**3**	**2**	**1**

The processing sequences and durationsof the three jobs are shown in

Table **2**,and thedue dates of all jobs are given in Table 3.

Table 2. Processing sequence / (duration) of the three jobs on the three machines

	Job 1	Job 2	Job 3
M/C 1	1 / (5)	2/ (10)	2/ (15)
M/C 2	0	1/ (20)	3/ (30)
M/C 3	2/ (40)	3 / (50)	1 / (60)

Table 3. Due date matrix of the three jobs

	Job 1	Job 2	Job 3
DUD	50	100	210

3.1.1 Results of Problems 1, 2 and 3

The resulting schedules of the first three problemsare the same as shown in Table 4. The schedule is drawn inGantt chart form (Figure 1). In this schedule, all jobs are supposed to be finished as soon as possible to fulfil the first priority objective of minimizing the makespan and the objective of minimizing total tardiness.

Table 4. The resultant production schedule of the first problem.

	Job 1	Job 2	Job 3
M/C 1	0 - 5	20 - 30	155 - 170
M/C 2		0 - 20	170 - 200
M/C 3	5 - 45	45 - 95	95 - 155
Completion Time	45	95	200

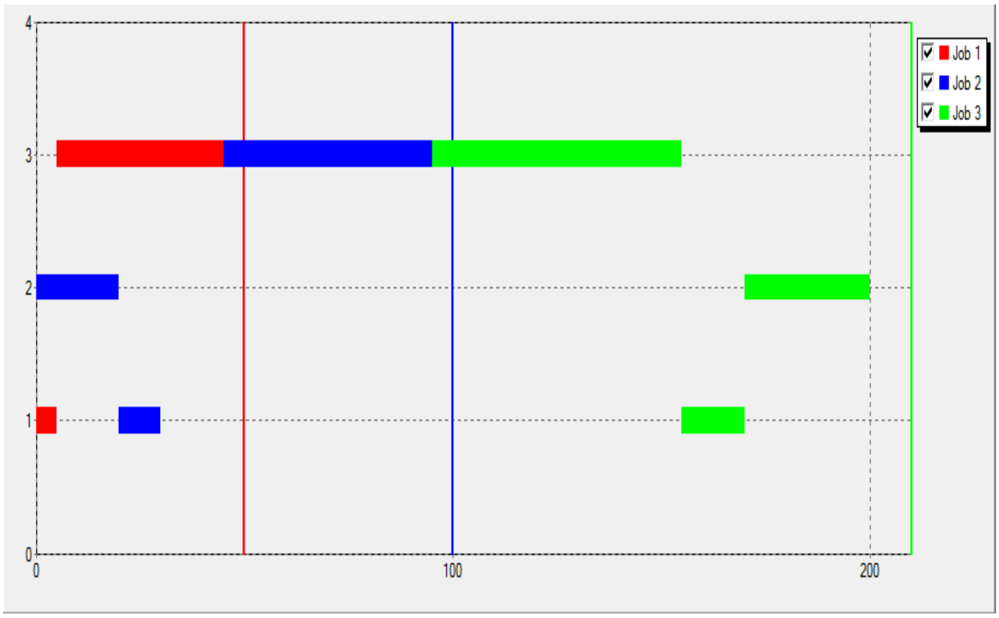

Figure 1. The Gantt chart of the first three problems.

The completiontime, tardiness, and earliness of the first three problems are given in Table 5.

Table 5. The completion time, tardiness, and earliness of the first three problems.

	Job 1	Job 2	Job 3
Completion Time	**45**	**95**	**200**
Tardiness	**0**	**0**	**0**
Earliness	**5**	**5**	**10**

Table 6shows the targets and values of the three objectives for the first three problems.

Table 6. The targets and values of the three objectives of the first three problems.

Objectives		Target	Value
1	Makespan	200	200
2	Total Tardiness	0	0
3	Total Earliness	20	20

In conclusion on Table 6, the optimal values of the first two objectives (makespan and total tardiness) are achieved.However, the third objective (total earliness) is not achieved. The first optimal values of the two objectives in the first problem, in which the first priority is to minimize the makespan and the second priority is to minimize the total tardiness, are achieved.The same result occured in the second problem, although the second priority is to maximize total earliness asthe makespan and tardiness are minimized asa combinedobjectives in this problem. Moreover, the same results are achieved for the third problem,in which the first priority is to minimize the total tardiness and the second is to minimize the makespan because of their consistency.

3.2.1Results of Problems4, 5 and 6

The resulting schedule of the last threeproblems are the same as shown in Table 7. The schedule is drawn in Gantt chart form (

Figure 2). In this schedule, all jobs are to be finished as close as possible to the due date to fulfil the first priority objective of minimizing total earliness and theindependent objective of minimizing total tardiness. The objective of minimizing the makespan is ignored in this problem.

Table 7. The production schedule of the last three problems.

	Job 1	Job 2	Job 3
M/C 1	5 - 10	20 - 30	160 - 175
M/C 2		0 - 20	180 - **210**
M/C 3	10 - **50**	50 - **100**	100 - 160
Completion Time	**50**	**100**	**210**

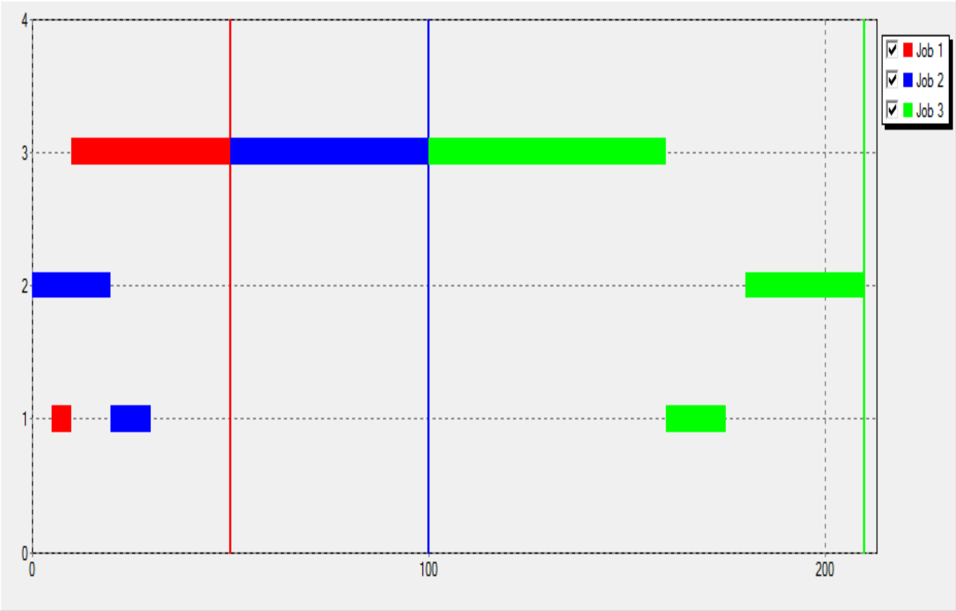

Figure 2. The Gantt chart of the last three problems

Table **8** shows the resultantcompletiontime, tardiness, and earliness of the last three problems.

Table 8. The Completion Time, Tardiness, and Earliness of the last three problems

	Job 1	Job 2	Job 3
Completion Time	**50**	**100**	**210**
Tardiness	**0**	**0**	**0**
Earliness	**0**	**0**	**0**

The targets and values of the three objectives are given in Table 9 for the last three problems.

Table 9. The targets and values of the three objectives for the last three problems.

Objectives		Target	Value
1	Total Earliness	0	0
2	Total Tardiness	0	0
3	Makespan	210	210

In conclusion on Table 9, the optimal values of the first two prior objectives (total earliness and total tardiness) are achieved however the third objective (makespan) is not achieved. The first optimal values of the two objectives for the fourth problem, in whichthe first priority is to minimize the total tardiness and the second priority is to minimize the total earliness, are achieved. The same resultsare achieved in the fifth problem, however, the second priority objective is inconsistent with the first objective. In addition, the first priority, minimizing total earliness and the second priority, minimizing total tardinessareconsistent in the sixth problem, andthe same results are achieved.

3.2 Effects Of Priority Ordering And Deviation

In this section, the effects of objective priority ordering and deviationare analysed and presented based onthirtyassumed problemswith six priority orders and five deviation values. Table 10presents the results of the thirty problems.

Table 10. Effects of objective deviation.

		Deviation				
		A	**B**	**C**	**D**	**E**
No.	**Objective**	**0 , 0,0**	**5 , 5, 5**	**10, 10, 10**	**15, 15, 15**	**20, 20, 20**
1	Comp. Time	45-95-**200**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**
	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	T. Earliness	5-5-10	0-0-0	0-0-0	0-0-0	0-0-0
2	Comp. Time	45-95-**200**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**
	T. Earliness	5-5-10	0-0-0	0-0-0	0-0-0	0-0-0
	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0

3	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	Comp. Time	45-95-**200**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**
	T. Earliness	5-5-10	0-0-0	0-0-0	0-0-0	0-0-0
4	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	T. Earliness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	Comp. Time	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**
5	T. Earliness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	Comp. Time	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**
	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
6	T. Earliness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	T. Tardiness	0-0-0	0-0-0	0-0-0	0-0-0	0-0-0
	Comp. Time	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**	50-100-**210**

In column A ofTable 10, thedeviation is zero;the optimal value of the first orderedobjective is achivedin all problems and consequently its consistent objectiveoptimal value is achieved. However, in column B,of 5% deviation for the first three problems, optimalmakespan values increased by 10 units (5% of 200).All objectives yieldedoptimal values because themakespanincreased from 200 to 210, whichallowed the model to achieve optimal valuesfor the other two objectives. Increasing the allowable deviation by more than 5% will not affect the results of the objectives,and optimal values are obtained at 5%.

3.3 Problems of10 Jobs on 10 Machines

Three problems of 10 jobs on 10 machines are studied to verify the ability of the model to solve large-scaleproblems as small-scaleproblems. The processing sequences and durations of the ten jobs are shown in Table 11and Table 12respectively. Moreover, the due dates of all jobs are given in Table 13.

Table 11. Processing sequence of the 10x10 problems.

	Job 1	Job 2	Job 3	Job4	Job5	Job6	Job7	Job8	Job9	Job10
M/C 1	-	1	10	4	7	10	7	1	6	10
M/C 2	4	2	3	1	8	7	-	2	8	9
M/C 3	2	3	2	2	9	8	1	3	4	8
M/C 4	3	4	1	3	-	9	6	4	7	7
M/C 5	7	9	5	5	6	3		5	1	6
M/C 6	8	8	6	10	5	4	4	6	9	1
M/C 7	5	7	7	9	4	1	3	7	10	2
M/C 8	6	6	8	8	3	2	5	-	3	3
M/C 9	1	5	9	7	2	5	2	9	2	4
M/C 10	9	-	4	6	1	6	8	8	5	5

Table 12. Duration matrix of the 10x10 problems.

	Job 1	Job 2	Job3	Job 4	Job5	Job6	Job7	Job8	Job9	Job10
M/C 1	0	19	13	19	13	11	12	13	12	12
M/C 2	15	17	17	14	18	11	0	19	17	4
M/C 3	18	14	16	16	9	0	12	17	18	11
M/C 4	16	12	18	11	0	0	13	10	0	0
M/C 5	14	16	12	18	11	16	12	13	11	19
M/C 6	16	12	18	19	10	0	13	14	3	9
M/C 7	17	18	10	16	16	12	17	18	14	16
M/C 8	14	9	11	11	18	0	14	0	12	14
M/C 9	17	14	17	10	16	13	12	14	16	12
M/C 10	13	0	20	13	12	12	16	16	12	9

Table 13. Due date matrix of the ten jobs.

	Job 1	Job 2	Job 3	Job4	Job5	Job6	Job7	Job8	Job9	Job10
DUD	170	140	250	230	240	150	280	260	190	270

The allowable deviations of the three objectives are assumedto equal 10%.The assumed priority orders of the three objectives in the above mentionedthree problems are shown in Table 14.The resulting schedules of the three problems are presented in Gantt chart form (Figure 3, Figure 4, and Figure 5).

Table 14. The priority orders of the three objectives for the three problems.

Exp. No.	Make Span	Total Tardiness	Total Earliness
1	1	3	2
2	3	1	2
3	2	3	1

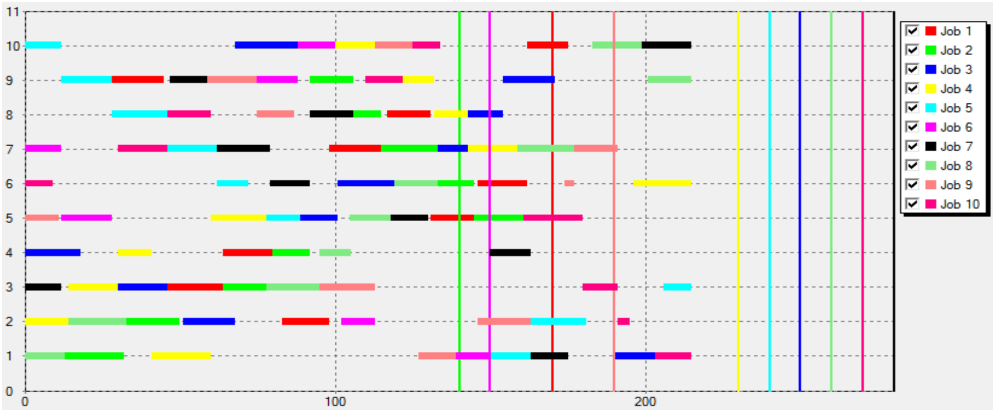

Figure 3. The Gantt chart of the first 10x 10 problem

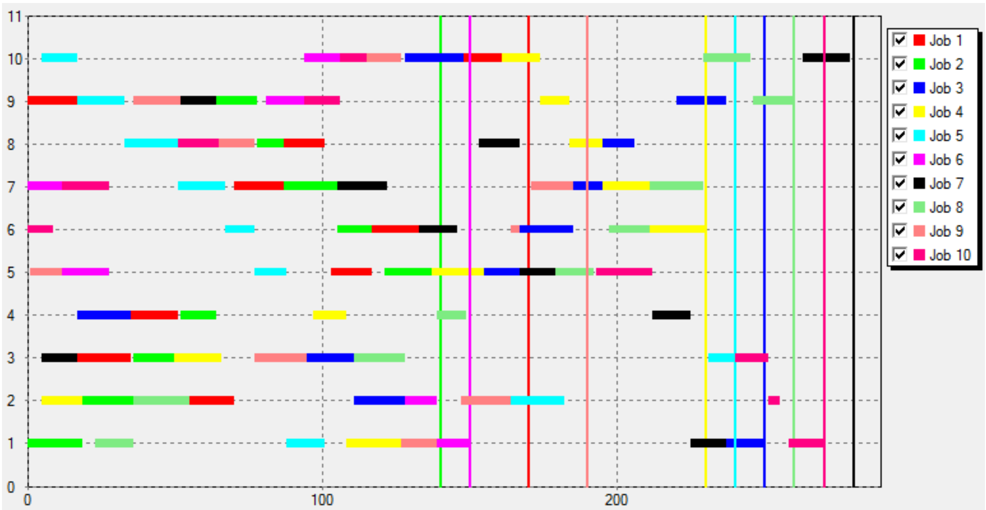

Figure 4. The Gantt chart of the second 10x10 problem

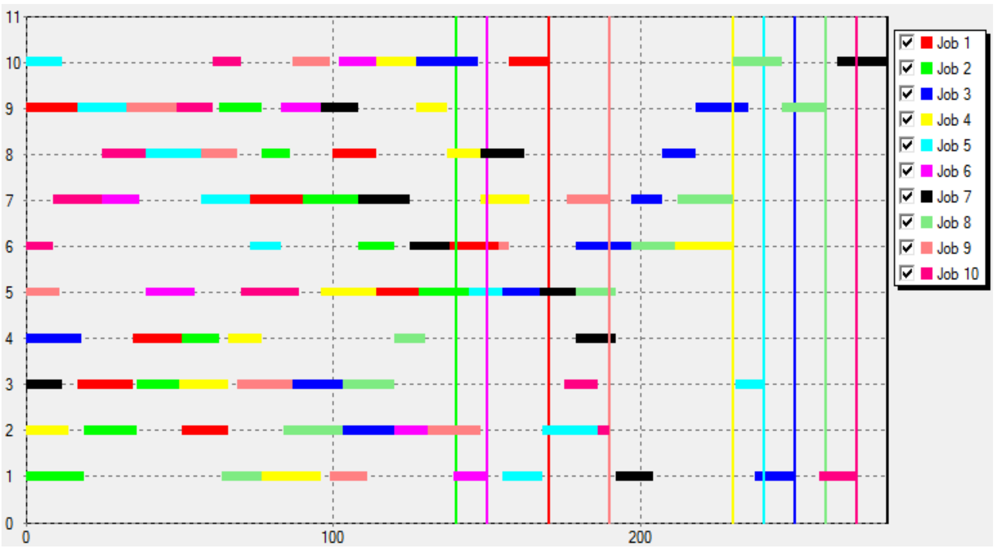

Figure 5. The Gantt chart of the third 10x10 problem

The finishing time, tardiness, and earliness of the three 10x10 problems are summarized in Table 15.

Table 15. The finishing time (F), tardiness (T), and earliness (E) of the three 10x 10 problems

	First Problem (F-T-E)			Second Problem (T-E-F)			Third Problem (E-F-T)		
	F	**T**	**E**	**T**	**E**	**F**	**E**	**F**	**T**
Job 1	175	5	0	0	9	161	0	170	0
Job 2	161	21	0	0	3	137	0	144	4
Job 3	203	0	47	0	0	250	0	250	0
Job 4	**215**	0	15	0	0	230	0	230	0
Job 5	**215**	0	25	0	0	240	0	240	0
Job 6	150	0	0	0	0	150	0	150	0
Job 7	**215**	0	65	0	1	**279**	0	**280**	0
Job 8	**215**	0	45	0	0	260	0	260	0
Job 9	191	1	0	0	5	185	0	190	0
Job 10	**215**	0	55	0	0	270	0	270	0
Obj.	**215**	**27**	**252**	**0**	**18**	**279**	**0**	**280**	**4**
CPU T.	**783 sec.**			**13 sec.**			**15 sec.**		

The resultant values of objectives of these three problems shown in Figure 6verify the accuracy and efficacy of the model. The makespanreaches the minimum optimal value in the first problembecause minimizing it is the first priority, unlike in the other two problems.Among the problems, the first problem exhibits the worsttotal earliness value because of its inconsistency with the first two prior objectives. Conversly, theoptimal value was reached in the third problembecause minimizing total tardiness was the first priority; however,this scheme produced the worst makespan value.Additionally, the best value of total tardiness occurred in the second problembecause it was given the highest priority.

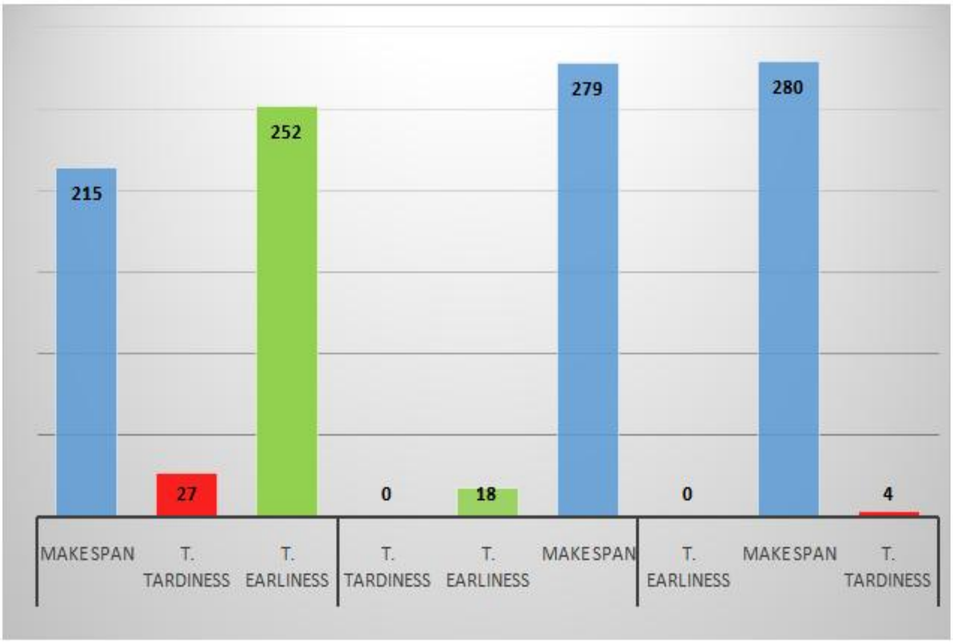

Figure 6. The resultant values of the three objectives for the three 10x10 problems.

The behaviour of the model can be clearly explained by comparing the objectives values with their targets. Figure 7shows the targets and values of the three objectives for the three 10x10 problems. In the first problem, the makespan and total tardiness valuesand the targets (the worest allowable value)are the same becausethe last priority objective of minimizing total earliness is inconsistent with the other objectives. The effect of the allowable deviation on the results of the first problem is presented in Figure 8. In the second problem, the optimal solution of the first objective (total tardiness) is zero, therefore, the allowable deviation of zero. However, the value of the second objective (total earliness) is maximized based on the target to improve the value of the final objective because they are inconsistent. In contrast, the makespan value is not equal to the associated target because it is consistent with the next objective, and increasing the makespan willnot improve the total tardiness.

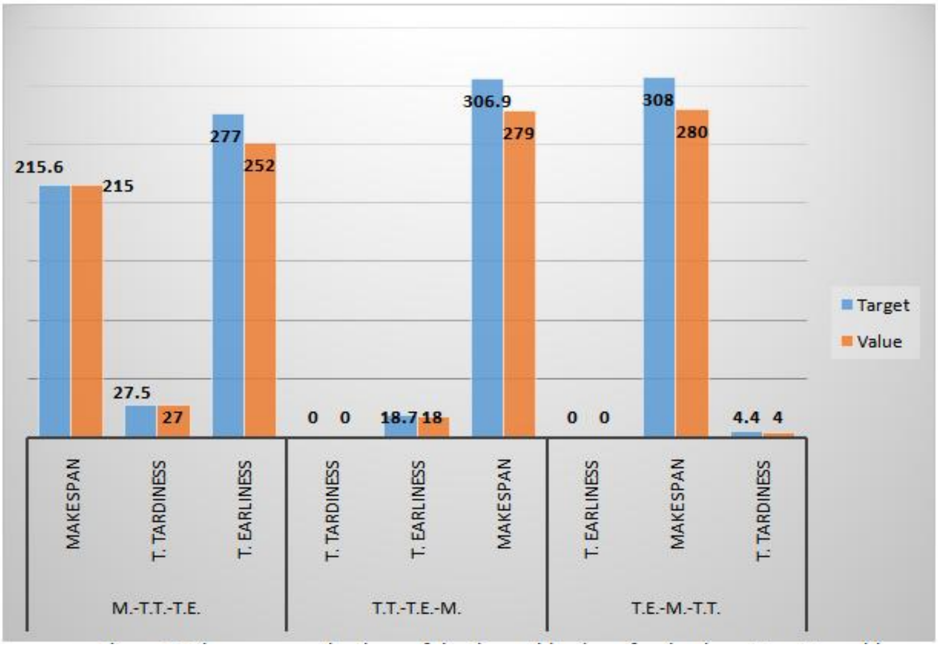

Figure 7. The targets and values of the three objectives for the three 10x10 problems

The first problem is resolved after changing the allowable deviation percentage to zero% instead of 10% to prove the model accuracy. Figure 8 presents the resultant schedules of the zero% deviation 10x10 problem in Gantt chart form.As shown in Figure 9, only the value of the first objective (makespan) is improved and the results of other objectives are worse, unlike in the 10% deviation solution; Notably, the value of 196 isincreased by 10% to allow for improvements in the subsequent objectives.

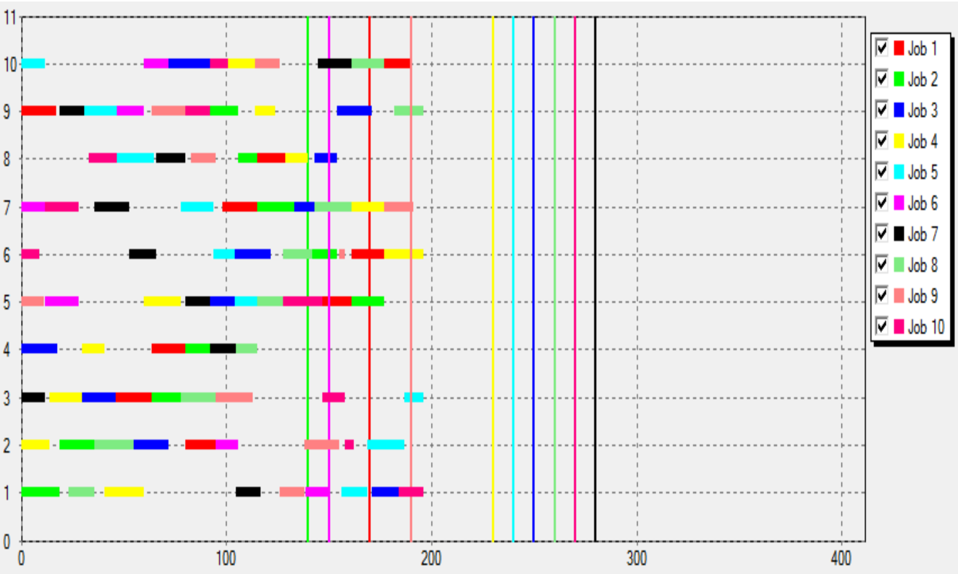

Figure 8. The Gantt chart of the zero deviation 10x10 problem

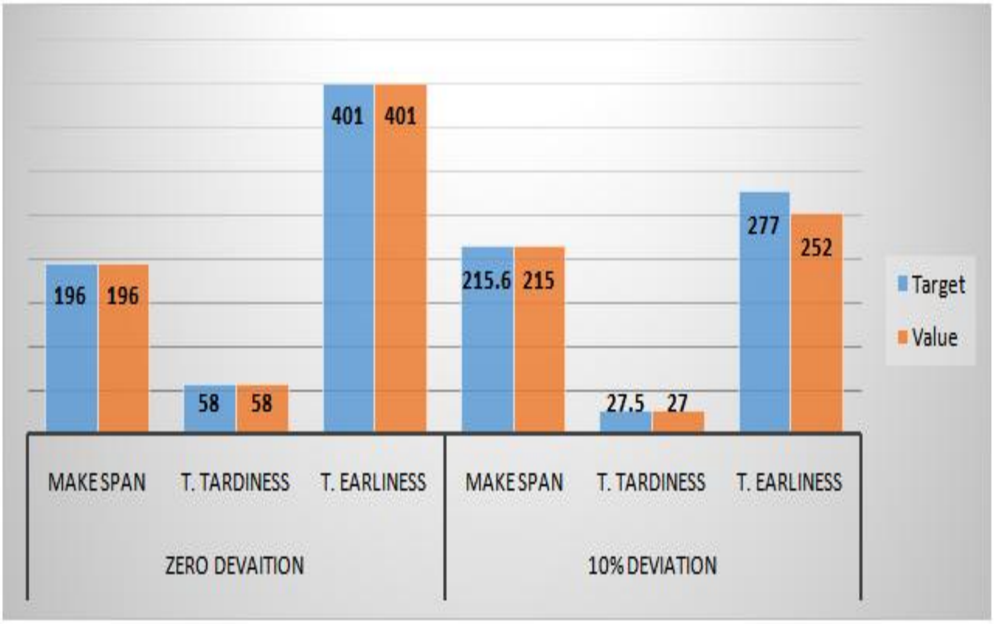

Figure 9. The targets and values of the three objectives for the first problem with zero% and 10% deviations.

IV. Conclusion

Objectives ordering is affected by the strategies of the firm. Prioritizing the makespan reduces the finishing time and consequently increases the utilization of machines. However prioritizing total earliness reduces the utilization of machines, increases customer satisfaction and aid JIT application.In this study, a Multi-Objective Job Shop Scheduling model using GP lexicographic procedure has been formulated and verified with makespan, total tardiness and total earliness criteria.Minimizing the makespan and total earliness areusuallyconsidered two inconsistent or conflicting objectives. Minimizing the makespan and total tardiness are usually two consistent objectives. However, minimizing the total earliness and total tardiness are usually two independent objectives.

The accuracy and efficacy of the developed model have been verified through the analysis and discussion of many problems of different sizes. The effect of objectives' deviations has been discussed and analyzed.

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