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Factorization and Pivotal Transform of Q-EP

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Abstract: In this paper the Factorization and Pivotal transform of q-EP matrices are discussed.
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I. Introduction

Throught we shall deal with $n \times n$ quaternion matrices: Let A^* denote the conjugate transpose of A. Any matrix $A \in H_{n\times n}$ is called q-EP. If $R(A) = R(A^*)$ and is called, q-EP$_r$ if A is q-EP and $rk(A) = r$, where $N(A),\ R(A)$ and $rk(A)$ denote the null space, range space and rank of A respectively. It is well known that sum and product of q-EP, Generalized Inverse Group Inverse and Reverse order law for q-EP, Bicomplex representation methods and application of q-EP matrices and Schur Complements in q-EP matrices [3-8].

1. FACTORIZATION OF q-EP

Throught this section, M is a $2n \times 2n$ matrix of the form $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$--------- (I)

With $\rho(M) = \rho(A) = r$. where A is $n \times n$ and D is $n \times n$ if M is q-EP. By [8, Theorem 1],

$N(A) \subseteq N(C)$, $N(A^*) \subseteq N(B^*)$, $D = CA^{\dagger}B$

Lemma 1.1

If M is q-EP$_r$ of the form(I) then there exists a $(p \times 2n - p)$ matrix X such that

$$M = \begin{pmatrix} A & AX \\ X^*A & X^*AX \end{pmatrix} \text{---------------------} \quad \text{(II)}$$

and A is q-EP$_r$.

Proof

Since M is of the form (I) and $\rho(A) = \rho(M)$. Hence there is an $(p \times 2n - p)$ matrix X such that $C = YA$ and $B = AX$ By [11,p.21]. Since M is q-EP, a is q-EP and $CA^{\dagger} = (A^{\dagger}B)^*$

$\Rightarrow YA = X^*A$

Also $D = CA^{\dagger}B = YAX = X^*AX$, therefore M is of the form(II)

Theorem 1.2

If M is q-EP$_r$ of the form(I) and A is q-EP$_r$, then M is a product of q-EP$_r$ matrices.

Proof

If M is q-EP$_r$ of the form(I) then it satisfies $N(A) \subseteq N(C)$, $N(A^*) \subseteq N(B^*)$, $D = CA^{\dagger}B$ then there exists X and Y such that

$C = YA, B = AX , D = CA^{\dagger}B = YAA^{\dagger}AX = YAX$

consider the matrices $S = \begin{pmatrix} A^{\dagger}A & AA^{\dagger}Y^* \\ YAA^{\dagger} & YAA^{\dagger}Y^* \end{pmatrix}$, $L = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$ and $T = \begin{pmatrix} A^{\dagger}A & AA^{\dagger}X \\ X^*A^{\dagger}A & X^*A^{\dagger}AX \end{pmatrix}$

By theorem [8, Theorem 1.8] S,L and T are q-EP$_r$ $CA^{\dagger} = (A^{\dagger}B)^{*}$

Also $(S)(L)(T) = \begin{pmatrix} A & AX \\ YA & YAX \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = M$

Thus, M is a product of S,L, and T are all q-EP$_r$ matrices. Therefore $M = SLT$.

Theorem 1.3

Let $L = \begin{pmatrix} E & F \\ G & F \end{pmatrix}$ be a $2n \times 2n$ matrix of rank r. if E is an $n \times n$ non-singular matrix then $L = S\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}T$, where S,T are q-EP$_r$ matrices.

Proof

$L = P\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}Q$, where P,Q are non-singular matrix. If we write

$P = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$, $Q = \begin{pmatrix} \hat{A}_1 & \hat{B}_1 \\ \hat{C}_1 & \hat{D}_1 \end{pmatrix}$, then $L = \begin{pmatrix} (A_1)(\hat{A}_1) & (A_1)(\hat{B}_1) \\ (C_1)(\hat{A}_1) & (C_1)(\hat{B}_1) \end{pmatrix}$ and $(A_1)(\hat{A}_1) = E$ is non-singular. Thus $A, \hat{A}$ are non-singular.

So, $\begin{pmatrix} A_1 \\ C_1 \end{pmatrix}$ and $\begin{pmatrix} \hat{A}_1 & \hat{B}_1 \end{pmatrix}$ have rank r.

Thus there is an $2n - r \times r$ matrix X and $r \times 2n - r$ matrix Y such that $XA_1 = C_1$ and $\hat{A}_1 Y = \hat{B}_1$, Put $S = \begin{pmatrix} A_1 & A_1X^{*} \\ XA_1 & XA_1X^{*} \end{pmatrix}$, $T = \begin{pmatrix} \hat{A}_1 & \hat{A}_1 Y \\ Y^{*}\hat{A}_1 & Y^{*}\hat{A}_1 Y \end{pmatrix}$

Now, $S = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}T = \begin{pmatrix} A_1 & A_1X^{*} \\ XA_1 & XA_1X^{*} \end{pmatrix}\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \hat{A}_1 & \hat{A}_1 Y \\ Y^{*}\hat{A}_1 & Y^{*}\hat{A}_1 Y \end{pmatrix} = L$ By [1,p.91]

Hence S,T are q-EP matrices.

Any matrix $A \in H_{2n\times 2n}$ of rank r is called a q-EP$_r$ matrix. If it has a principal $r \times r$ non-singular matrix.

Lemma 1.4

Let M be a $2n \times 2n$ matrix of order r. If M is a P$_r$ matrix then M is a product of q-EP$_r$ matrices.

Proof

Let M be a $2n \times 2n$ matrix of order r having E as a principal $r \times r$ non-singular sub-matrix there is a permutation matrix P such that

$PMP^{T} = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$, by theorem 1.3 $\begin{pmatrix} E & F \\ G & H \end{pmatrix} = S\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}T$, where S,T are q-EP$_r$ matrices.

Hence, $PMP^{T} = S\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}T$, $M = P^{T}S\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}TP$, $M = (P^{T}SP)P^{\dagger}\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}P(P^{\dagger}TP)$

Since S,T are q-EP$_r$ matrices, $P^{T}SP$ and $P^{\dagger}TP$ are q-EP matrices. Thus, M is a product of q-EP matrices.

Remark 1.5

The converse of theorem (1.4) need not be true.

Example 1.6

Let $A=\begin{pmatrix}0&0&i\\0&1&j\\-i&-j&1\end{pmatrix}, B=\begin{pmatrix}0&k&0\\-k&1&0\\0&0&1\end{pmatrix}, C=\begin{pmatrix}0&0&k\\0&1&0\\-k&0&1\end{pmatrix}$

Where A,B,C are q-EP matrices of rank 3 but $ABC=\begin{pmatrix}j&0&i\\-i&1&1+j\\-k&0&-j+1\end{pmatrix}$ has rank 3, does not have a P_3 Matrices. More over, ABC is not q-EP.

Lemma 1.7

Let $A=\begin{pmatrix}E&F\\G&H\end{pmatrix}$ be a q-EP matrix is an $r\times r$ matrix and $\begin{pmatrix}E&F\end{pmatrix}$ has rank r, then E is non-singular.

Proof

$$\begin{pmatrix}I_r&0\\0&0\end{pmatrix}\begin{pmatrix}E&F\\G&H\end{pmatrix}=\begin{pmatrix}E&F\\0&0\end{pmatrix}$$ where I_r is the $r\times r$ identity matrix.

$\begin{pmatrix}E&F\\G&H\end{pmatrix}\begin{pmatrix}I_r&0\\0&0\end{pmatrix}=\begin{pmatrix}E&0\\G&0\end{pmatrix}$ has rank r. By [11, P.52] E has rank r. Thus E is non-singular.

Lemma 1.8

Let A and B be $2n\times 2n$ q-EP matrices. If AB has rank r, then AB is unitarily similar to a P_r matrix.

Proof

Since A is q-EP_r, there is a unitary matrix U such that A is unitarily similar to a diagonal block q-EP_r matrix $\begin{pmatrix}D&O\\O&O\end{pmatrix}$ where D is a $r\times r$ non-singular matrix

$$A=U\begin{pmatrix}D&O\\O&O\end{pmatrix}U^*, \Rightarrow \text{put } U^*(B)U=\begin{pmatrix}E&F\\G&H\end{pmatrix}$$ where E is $r\times r$ matrix

Then, $U^*(A)(B)U=\begin{pmatrix}D&O\\O&O\end{pmatrix}\begin{pmatrix}E&F\\G&H\end{pmatrix}, U^*(AB)U=\begin{pmatrix}DE&DF\\O&O\end{pmatrix}$ has rank r.

Thus $(U^*AU)(U^*BU)=\begin{pmatrix}E&F\\G&H\end{pmatrix}\begin{pmatrix}D&O\\O&O\end{pmatrix}$

$$\Rightarrow U^*ABU=\begin{pmatrix}ED&0\\GD&0\end{pmatrix} therefore, GD=O \text{ Hence } G=O$$

E is Non-Singular. Applying lemma 1.3. A is a product of q-EP_r matrices.

Remark 1.10

The condition on $\rho(A)=r$ is essential If $\rho(A)\neq r$ then theorem(1.9) fails.

For example,

Let $A=\begin{pmatrix}0&i\\-i&1\end{pmatrix}, B=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ Here $\rho(A)=1, \rho(B)=0$, B is q-EP_0 $AB=\begin{pmatrix}0&0\\0&0\end{pmatrix}$ is q-EP_0,Here $B=AB$. Hence the statement of (1.9) fails.

Theorem 1.11

Let $M=\begin{pmatrix}A&B\\C&D\end{pmatrix}, L=\begin{pmatrix}P&Q\\R&S\end{pmatrix}$ be q-EP_r matrices and ML be of rank r. Then the following are equivalent.

(i) ML is q-EP_r

(ii) AP is q-EP_r and $CA^{\dagger} = RP^{\dagger}$

(iii) AP is q-EP_r and $A^{\dagger}B = P^{\dagger}Q$

Proof

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, L = \begin{pmatrix} P & Q \\ R & S \end{pmatrix}$$

$$(M)(L) = \begin{bmatrix} A & AX \\ X^*A & X^*AX \end{bmatrix}\begin{bmatrix} P & PY \\ Y^*P & Y^*PY \end{bmatrix} = \begin{pmatrix} AZP & AZPY \\ X^*AZP & X^*AZPY \end{pmatrix}, Z = 1 + XY^*$$

Clearly, $N(AZP) \subseteq N(X^*AZPY)$

$$N(AZP)^* \subseteq N(X^*AZPY)$$

Schur complements of AZP in ML,

$$(ML / AZP) = (X^*AZPY) - (X^*AZP)(AZP)^{\dagger}(AZP) = 0 \quad , \ \rho(AZP) = \rho(ML) = r$$

Hence by theorem "Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ with $\rho(M) = \rho(A) = r$, then M is q-EP_r and $CA^{\dagger} = (A^{\dagger}B)^*$".

A and P are both q-EP_r matrices

$$CA^{\dagger} = (A^{\dagger}B)^*, RP^* = (P \dagger Q)^*$$

$$R(AZP) \subseteq R(A)$$

$$R(AZP)^* \subseteq R(P^*) = R(P) \qquad \text{(Since P is q-EP)}$$

and $\rho(AZP) = \rho(A) = \rho(P) = r$

Hence, $R(AZP) = R(A); R(AZP)^* = R(P)$

$$(AZP)(AZP)^{\dagger} = (A)(A)^{\dagger}$$

$$(AZP)^{\dagger}(AZP) = (P)(P)^{\dagger}$$

ML is q-EP_r $\Leftrightarrow$ (M)(L) is EP_r

$\Leftrightarrow$ AZP is EP_r

$$(X^*AZP)(AZP)^{\dagger} = (AZP)^{\dagger}(AZPY)^*$$

$$\Leftrightarrow R(AZP) = R(AZP)^*$$

$$X^*(A)(A)^{\dagger} = Y^*(P)(P)^{\dagger} \Leftrightarrow R(A) = R(P)$$

and by $(AZP)(AZP)^{\dagger} = AA^{\dagger}, (X^*A)(A^{\dagger}) = (Y^*P)(P^{\dagger})$

Since A and P are both q-EP_r matrices,

$\Leftrightarrow$ AP is q-EP_r, $CA^{\dagger} = RP^{\dagger}$

$\Leftrightarrow$ AP is q-EP_r and $CA^{\dagger} = RP^{\dagger}$

$\Leftrightarrow$ AP is q-EP_r and $(A^{\dagger}B)^* = (P^{\dagger}Q)^*$

$\Leftrightarrow$ AP is q-EP_r and $(A^{\dagger}B) = P^{\dagger}Q$

Thus, ML is q-EP_r $\Leftrightarrow$ AP is q-EP_r and $A^{\dagger}B = P^{\dagger}Q$

II. PIVOTAL TRANSFORM ON Q-EP MATRICES

Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ then principal re-arrangement of square matrix M (i.e) P^TMP, where P is a permutation matrix, $P^TMP = \begin{pmatrix} D & C \\ B & A \end{pmatrix}$, where p is permutation matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Let us consider a system of linear equations, $MZ = T$, where $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ satisfying $N(A) \subseteq N(C), N(A^*) \subseteq N(B^*)$.

If z and t are partitioned conformably as $z = \begin{bmatrix} x \\ y \end{bmatrix}$ and $t = \begin{bmatrix} u \\ v \end{bmatrix}$, then $Ax + By = u$, $Cx + Dy = v$ Then by [10,p.21] we can solve for x and v as $x = A^{\dagger}u^{-}A^{\dagger}$ By, $v = CA^{\dagger}u + (D - CA^{\dagger}B)y$. Thus a matrix $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ satisfying $N(A) \subseteq N(C), N(A^*) \subseteq N(B^*)$ can be transformed into the matrix

$$\hat{M} = \begin{pmatrix} A^{\dagger} & -A^{\dagger}B \\ CA^{\dagger} & (M/A) \end{pmatrix} \text{--} \quad (1)$$

$\hat{M}$ is called a principal pivot transform of M.

Lemma 2.1

Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ with $N(A) \subseteq N(C)$, $N(D) \subseteq N(B)$ then the following are equivalent.

(i) M is q-EP, $N(M/A) \subseteq N(B)$, $N(M/D) \subseteq N(C)$

(ii) A and M/D are q-EP and D and (M/A) are q-EP

Further, $N(A) = N(M/D) \subseteq N(B^*)$ and $N(D) = N(M/A) \subseteq N(C^*)$

Proof

$(i) \Leftrightarrow (ii)$ Since M is q-EP, $N(A) \subseteq N(C)$, $N(M/A) \subseteq N(B)$ By theorem [8, Theorem1], A is q-EP and M/A is q-EP, $N(A^*) \subseteq N(B^*)$ and $N(M/A)^*) \subseteq N(C^*)$. Since A is q-EP, $N(A^*) = N(A)$ (By definition of q-EP).

Therefore $N(A) = N(B^*)$ since M is q-EP, M is EP, implies the principal rearrangement $P^T MP = \begin{pmatrix} D & C \\ B & A \end{pmatrix}$ is also EP. Further $N(D) \subseteq N(B)$ and $N(D) \subseteq N(B)$ and $N(M/D) \subseteq N(C)$ holds hence by theorem [6, Theorem 1], D is EP (M/D) is EP.

$N(D^*) \subseteq N(C^*)$ and $N(M/D) \subseteq N(B^*)$

Thus we have, D is q-EP, (M/D) is q-EP.

$N(D^*) \subseteq N(C^*)$ and $N(M/D) \subseteq N(B^*)$

Since D is q-EP, by definition $N(D^*) = N(D)$.

Thus $N(D) \subseteq N(C^*)$

Since the relations, $N(A) \subseteq N(C)$, $N(A^*) \subseteq N(C), N(A^*) \subseteq N(B^*)$, $N(M/A) \subseteq N(B)$ and $N(M/A)^* \subseteq N(C^*)$ holds for A.

According to the assumption and from the definition

$$M^{\dagger} = \begin{pmatrix} A^{\dagger} + (A^{\dagger})(B)(M/A)^{\dagger}(A^{\dagger}) & -(A)^{\dagger}B(M/A)^{\dagger} \\ -(M/A)^{\dagger}C(A)^{\dagger} & (M/A)^{\dagger} \end{pmatrix} \text{------------------------------} \ (2)$$

Using $C = (M/A)(M/A)^{\dagger}$ and $B = (A)(A)^{\dagger}(B)$

$$(M^{\dagger})(M) = \begin{pmatrix} (A)(A)^{\dagger} & 0 \\ 0 & (M/A)(M/A)^{\dagger} \end{pmatrix} \text{------------------------------} \ (3)$$

Since the relations, $N(D)\subseteq N(C)$ and $N(M/D)\subseteq N(B^*)$ holds for d, according to the assumptions by theorem.

$$(M)^\dagger = \begin{pmatrix} (M/D)^\dagger & -(A)^\dagger B(M/A)^\dagger \\ -(D)^\dagger C(M/D)^\dagger & (M/A)^\dagger \end{pmatrix} \quad \text{------------------------------------} \quad (4)$$

$C=(D)(D)^\dagger C, C=DD^\dagger C$ and $B=(A)(A)^\dagger(B), B=AA^\dagger B$ in (3)

$$(M)(M^\dagger) = \begin{pmatrix} (M/D)(M/D)^\dagger & 0 \\ 0 & (M/A)(M/A)^\dagger \end{pmatrix}$$

Comparing (2) and (4) $(A)(A^\dagger)=(M/D)(M/D)^\dagger \Leftrightarrow AA^\dagger=(M/D)(M/D)^\dagger$

Since A and (M/D) are q-EP, $A^\dagger A=(M/D)^\dagger(M/D)$ $A^\dagger A=(M/A)^\dagger(M/D)$

Thus, $N(A)=N(M/D)$

Similarly, we can obtain the expressions for $M^\dagger M$, comparing $D^\dagger D=(M/A)^\dagger(M/A)$

$\Leftrightarrow N(D)=N(M/A)$

$(ii)\Rightarrow(i)$: $N(M/A)\subseteq N(B)$ follows directly from $N(M/A)=N(D)\subseteq N(B)$

Similarly, $N(M/D)\subseteq N(C)$ follows $N(M/D)=N(A)\subseteq N(C)$

Now, A is q-EP and (M/A) is q-EP satisfying the relations $N(A)\subseteq N(C)$, $N(A^*)\subseteq N(B^*)$, $N(M/A)\subseteq N(B)$ and $N(M/A)^*\subseteq N(C)^*$

Hence by theorem [8, Theorem 1] . Therefore M is q-EP. Thus (i) holds.

Theorem 2.2

Let $M=\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be a q-EP$_r$ matrix, $N(A)\subseteq N(C)$, $N(D)\subseteq N(B)$, $N(M/A)\subseteq N(B)$ $N(M/A)\subseteq N(B)$ and $N(M/A)\subseteq N(C)$. Then the following are hold.

(i) Principal sub-matrix A is q-EP and principal sub matrix D is q-EP

(ii) The Schur Complement (M/A) is q-EP

(iii) Each principal pivot transforms of M is q-EP

Proof

(i) and (ii) are consequence of lemma 2.1 (iii); By Lemma 2.1, M satisfies $N(A)\subseteq N(C)$ $N(A^*)\subseteq N(B^*)$ hence by pivoting the block A, the principal pivot transform $\hat{M}$ of M is of the form.

$$\hat{M} = \begin{pmatrix} (A)^\dagger & -(A)^\dagger(B) \\ (C)(A)^\dagger & (M/A) \end{pmatrix}, \quad \hat{M} = \begin{pmatrix} A^\dagger & -A^\dagger \\ CA^\dagger & (M/A) \end{pmatrix}$$

In $\hat{M}$, $N(A^\dagger)\subseteq N(CA^\dagger)$, $N(A^\dagger)^*\subseteq N(CA^\dagger)^*$

Further, $(\hat{M}/A^\dagger)=(M/A)+(CA^\dagger)(A^\dagger)^\dagger(A^\dagger B)=(M/A)+CA^\dagger AA^\dagger B=(M/A)+CA^\dagger B$

$(\hat{M}/A^\dagger)=D$

By the assumption, $N(\hat{M}/A^\dagger)=N(D)$ which implies $N(\hat{M}/A^\dagger)=N(D)\subseteq N(B)$.

From Lemma 2.1, A is q-EP and D is q-EP. Therefore, $A^\dagger$ is q-EP and $(\hat{M}/A^\dagger)$ is q-EP.

Hence, $D=(\hat{M}/A^\dagger)$

Also, $N(\hat{M}/A^\dagger)^*=N(D^*)$, $N(\hat{M}/A^\dagger)^*=N(D^*)\subseteq N(C^*)$

Now applying theorem [8, Theorem 2.1]

Now $r = \rho(M) = \rho(A) + \rho(M/A)$ [2, Theorem 1]

$$= \rho(A^{\dagger}) + \rho(D) \quad \text{(Lemma 2.1)}$$

$$= \rho(A^{\dagger}) + \rho(\hat{M}/A^{\dagger})$$

$$= \rho(\hat{M}) \quad \text{[2, Theorem 1]}$$

Thus $\hat{M}$ is q-EP$_r$. Similarly, under the conditions given on M, M can be transformed to its principal pivot transform by pivoting the block D without changing the rank.

Remark 2.3

For $K(i) = i$, (the identity transposition), theorem (2.2) reduced to Theorem [9, Theorem]

Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, then $\rho(M) \geq \rho(A) + \rho(M/A)$ with equality if and only if $N(M/A) \subseteq N(IAA^{\dagger})B$

$$N(M/A)^{*} \subseteq N(I - A^{\dagger}A)C^{*} \text{ and } (I - AA^{\dagger})B(M/A) \subseteq (I - A^{\dagger}A) = 0$$

Remark 2.4

In the special case when M is non-singular with A and D non-singular, then the conditions $N(A) \subseteq N(C)$ and $N(D) \subseteq N(B)$. Automatically hold (M/A) and (M/D) are non-singular by [2, Theorem 1].

Further, $\rho(\hat{M}) = \rho(A) + \rho(D)$. Hence it follows that each principal pivot transform of M is non-singular. We note that the non-singularity of $\hat{M}$ need not imply M is non-singular.

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