MHD Stagnation Point Flow over a Stretching Sheet with the Effects of Thermal Radiation and Slip Conditions

Chenna Sumalatha¹, Bandari Shankar², T. Srinivasulu³

¹Research scholar in Department of Mathematics, Osmania University, Hyderabad, India
 ²Department of Mathematics, Osmania University, Hyderabad, India
 ³Department of Mathematics, M. V. S GDC, Mahabubnagar, Telangana, India
 Corresponding Author: Chenna Sumalatha

Abstract : The present study deals with the effects of MHD stagnation point flow over a stretching sheet with thermal radiation and slip boundary conditions. Using similarity transformations the governing equations are reduced to coupled nonlinear ordinary differential equations. The effective numerical technique Keller Box method is used to solve these equations. The variations in velocity, temperature profiles are presented with the various values of Magnetic parameter M, Suction parameter S, Grashof number Gr, Prandtl number Pr, Radiation parameter R, Heat generation parameter Q, Heat absorption parameter d, Velocity ratio parameter ε , Velocity Slip parameter A and Thermal Slip parameter B. We observed that velocity profile decreases with the increasing values of velocity slip parameter and also temperature increases when thermal slip parameter values increases.

Keywords - MHD, Stagnation point, Thermal Radiation, Keller Box Method, Stretching sheet, Slip boundary conditions

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I. Introduction:

A large amount of research has been done on stagnation-point flows. Due to its enormous applications in industries such as cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown and hydrodynamic processes, stagnation point flows attracted many researchers. In fluid mechanics, a point where the local velocity of the fluid becomes zero is called a stagnation-point. Stagnation points exist at the surface of objects in the flow field, where the fluid is brought to the rest by the object. Recently, Nazar et al. [1] discussed unsteady two-dimensional stagnation point flow of an incompressible viscous fluid over a deformable sheet. They discussed the analysis when the flow is started impulsively from rest and the sheet is suddenly stretched in its own plane with a velocity proportional to the distance from the stagnation point. They used Keller-box method to solve the unsteady boundary layer equations. Ishak et al [2] demonstrated the stagnation-point flow and heat transfer over a shrinking sheet in a micropolar fluid. Wang [3] investigated the shrinking rate due to an increase in the boundary layer thickness. Chen [4] presented the unsteady mixed convection flow over a stretching sheet in the presence of velocity and thermal slips near the stagnation-point. A good amount of literature has been generated on the stagnation-point flows towards a stretching or shrinking sheet [5-11].

In all investigations mentioned above, the flow field obeys the no slip conditions. It is known that no slip conditions state that the moving fluid in contact with a solid body will not have any velocity relative to the body at the contact surface (Prabhakara and Deshpande [12]). The no-slip boundary condition is the basic principle of the Navier–Stokes equations theory. But there are situations where such a condition is not appropriate. Especially in the case of non-Newtonian liquids, the no-slip condition is inadequate. On melting, many polymers exhibit microscopic wall slip, which is governed by a nonlinear and monotone relation between the slip velocity and traction. Navier himself in 1827 suggested a slip boundary condition in terms of shear stress. After Navier, much research has been conducted on the extension of his work. Hence, the problem of slip on a stretching sheet for different cases of fluid has been analyzed by different researchers. The earlier studies that took into account the slip boundary condition over a stretching sheet were conducted by Andersson [13]. He gave a closed form solution of a full Navier–Stokes equations for a magneto hydrodynamics flow over a stretching sheet. Velocity and thermal slip conditions are adequate for the flow of liquids at the micro-scale level especially in view of the lack of data on the thermal accommodation coefficient. Momentum and heat

transfer in laminar boundary layer with slip effects are discussed by Martin and Boyd [14]. A very good collection of articles on this topic can be found in [14-18].

The study of boundary layer flows with the effects of heat and mass transfer over a stretching sheet has various applications in industrial and engineering processes. Some of its applications are manufacture and extraction of polymer and rubber sheets. The flow due to stretching of a flat surface was first investigated by Crane [19]. The heat and mass transfer occurring in the laminar boundary layer on a linearly accelerating surface with temperature dependent heat source subject to suction or blowing was investigated by Acharya et al. [20]. The stretching problem of an incompressible fluid over a permeable wall was studied by Magyari and Keller [21]. The steady boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet was analyzed by Bhattacharyya [22]. The mixed convection MHD flow of a Casson nanofluid over a nonlinear permeable stretching sheet with viscous dissipation was proposed by Prabhaker [23]. Yahaya and Simon [24] investigated the theoretical influence of buoyancy and thermal radiation on MHD flow over a stretching porous sheet.

Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. The study of magnetohydrodynamics has wide applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter. The effect of external magnetic field on the MHD flow over a stretching sheet was examined by Pavlov [25]. Dulal and Hiranmoy [26] explained the effects of variable thermal conductivity, soret and dufour on MHD non-Darcy mixed convection heat and mass transfer over a nonlinear stretching sheet in the presence of thermal radiation, viscous dissipation, non-uniform heat source/sink and first-order chemical reaction. Chamkha [27] discussed the thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation was studied by Swati Mukhopadhyay [28]. The Keller-Box method introduced by Keller [29] is one of the best numerical method basically it's a mixed finite volume method which consists in taking the average of a conservation law and of the associated constitutive law at the level of the same mesh cell. Sarif [30] worked the numerical solution of the steady boundary layer flow and heat transfer over a stretching sheet with Newtonian heating by using Keller box method.

Motivated by the above literature, the main aim of the study deals with the analyzation of the effects of thermal radiation and slip parameters on MHD stagnation point flow of a stretching sheet. The basic governing equations are converted into ordinary differential equations by applying suitable similarity transformations and those equations were solved numerically by using finite difference method called as the Keller Box method.

II. Mathematical Formulation:

Consider a two dimensional steady MHD stagnation point flow of incompressible, electrically conducting, viscous flow over a stretching sheet. The velocity of the stretching sheet is $u_w(x) = ax$ (where a > 0 is the constant acceleration parameter). The x-axis is taken along the stretching sheet and y is the coordinate normal to the surface. The fluid is electrically conducting under the influence of magnetic field B(x) normal to the stretching sheet. The induced magnetic field is neglected due to assume magnetic Reynolds number is small. Assume that wall temperature is T_w and wall suction is $v_{w.}$. The ambient temperature of the flow is T_{∞} . The thermal physical properties of the flow assumed to be constant. The pressure gradient and external forces are neglected. All the fluid properties are assumed to be constant.

Under the above assumptions, MHD stagnation point flow over a stretching sheet with the effect of thermal radiation and slip conditions are governed by the following equations:

The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

The momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_{\infty}\frac{\partial U_{\infty}}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B^2}{\rho}(u - U_{\infty}) + g\beta_T(T - T_{\infty})$$
(2)

The energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_o}{\rho c_p} (T - T_{\infty}) + \frac{\beta^* u}{\rho c_p} (T_{\infty} - T) + T_{\infty}^3 \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$
(3)

Using the Rosseland approximation $q_r = -(\frac{4\sigma^*}{3k^*})\frac{\partial T^4}{\partial y}$ is obtained where σ^* is the Stefan-Boltzmann constant and k^* is the absorption coefficient. We presume that the temperature variation within the flow is such that T^4 may

be expanded in a Taylor's series. Expanding T^4 about T_{∞} and neglecting higher-order terms we get T^4 = $4T_{\infty}^{3}T - 3T_{\infty}^{4}$, then q_r becomes

$$q_{r} = -\frac{16T_{\omega}^{*}\sigma^{*}}{3k^{*}}\frac{\partial T}{\partial y}$$

Substitute q_{r} value in (3) we obtain
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^{2}T}{\partial y^{2}} + \frac{Q_{o}}{\rho c_{p}}(T - T_{\omega}) + \frac{\beta^{*}u}{\rho c_{p}}(T_{\omega} - T) + T_{\omega}^{3}\frac{\partial^{2}T}{\partial y^{2}} + \frac{16\sigma^{*}T_{\omega}^{3}}{3\rho c_{p}k^{*}}\frac{\partial^{2}T}{\partial y^{2}}$$
(4)
The associated boundary conditions are

The associated boundary conditions are

$$At \ y=0: \qquad u=u_{w}+L\frac{\partial u}{\partial y}; \qquad v=v_{w}; \qquad T=T_{w}+K_{1}\frac{\partial T}{\partial y};$$
(5)
$$At \ y\to\infty: \quad U_{\infty}\to0; \qquad T\to T_{\infty};$$

Here
$$u_w = ax$$
, $T = T_{\infty} + A_0 x$ and $U_{\infty} = bx$

Where u, v are velocity components along the x, y axis respectively. $U_{\infty} = bx$ is the straining velocity of the stagnation-point flow with b > 0 being the straining constant, σ is the electrical conductivity, g is the acceleration due to gravity, ρ is the fluid density, β_{T} is the coefficient of thermal expansion, B is the magnetic field, V is the kinematic viscosity, α is the thermal diffusivity of the fluid, Q_0 is the heat generation, c_p is the specific heat at constant pressure, σ^* is the Stepan Boltzmann constant, k^* is the mean absorption coefficient, L is the velocity slip factor, K_1 is the thermal slip factor. When $L = K_1 = 0$ the no-slip condition components along x-axis and y-axis respectively.

Using similarity transformations

$$\eta = y \sqrt{\frac{a}{v}} x \quad u = axf^{1}(\eta)' \quad \varphi = \sqrt{avx} f(\eta) \quad \theta(\eta) = \frac{T - T_{w}}{T_{w} - T_{\infty}}$$
(6)

Where ψ denotes stream function and is defined as $u = \frac{\partial \psi}{\partial v}$, $v = -\frac{\partial \psi}{\partial x}$ and $f(\eta)$ is a

dimensionless stream function, θ is dimensionless temperature function and η is similarity variable. After similarity transformations, the governing equations (2)-(4) are reduced as follows:

$$f''' + ff'' - (f')^{2} + Mf' + Gr\theta + L^{2} = 0$$

$$\left(\frac{R+1}{\Pr}\right)\theta'' + f\theta' + d\theta - (1+Q)f'\theta = 0$$
(8)

Where R is the radiation parameter, Gr is the Grashof number, M is the magnetic parameter, Q is the heat generation, d is the heat absorption.

The associative boundary conditions becomes

$$f'(0) = 1 + Af''(0)$$
, $f(0) = S$, $\theta(0) = 1 + B\theta'(0)$, $as \eta \to 0$ (9)

$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \qquad as \ \eta \to \infty$$
 (10)
Where

$$M = \sqrt{\frac{\sigma}{\rho a}} B_0 \quad , \quad A = L \sqrt{\frac{a}{\nu}} \quad , \quad B = K_1 \sqrt{\frac{a}{\nu}} \quad , \quad \Pr = \frac{\nu}{\alpha} \quad , \quad S = \frac{-\nu_w}{\sqrt{a\nu}}$$
$$R = \frac{16\sigma^* T^3}{3kk^*} \quad , \quad \nu = \frac{k}{(\rho c)_f} \quad , \quad Gr = g\beta \frac{(T_w - T_w)}{a^2 x} \quad , L = \frac{a}{b}$$
(11)

Here A, B, S, R, L are velocity slip parameter, thermal slip parameter, suction parameter, radiation parameter and velocity ratio parameter respectively .

III. Numerical Procedure:

The boundary value problem (7)–(8) is solved by a second order finite difference scheme known as the Keller Box method [29]. The numerical solutions are obtained in four steps as follows:

- Reduce the equations to a system of first order equations;
- write the difference equations using central differences;
- linearize the algebraic equations by Newton's method, and write them in matrix-vector form; and
- solve the linear system by the block tri-diagonal elimination technique.

The step size $\Delta \eta$ and the position of the edge of the boundary layer η_{∞} are to be adjusted for different values of the parameters to maintain accuracy. For numerical calculations, a uniform step size of $\Delta \eta = 0.01$ is found to be satisfactory and the solutions are obtained with an error tolerance of 10^{-6} in all the cases.

IV. Result And Discussion:

The non-linear ordinary differential equations Eqs. (8) – (9) with the boundary conditions (10) were solved numerically by Keller Box method. The computation have been carried out for different values of governing parameters viz. Magnetic parameter M, Suction parameter S, Grashof number Gr, Prandtl number Pr, Velocity ratio parameter ε , Thermal radiation parameter R, Heat generation coefficient Q, Heat absorption coefficient d, Velocity slip parameter A, Thermal slip parameter B. The velocity and temperature profiles for different governing parameters have examined. The results obtained in the study are compared with the existing literature and found in good agreement which is presented in the Table 1. Comparison of wall temperature gradient – $\theta'(0)$ for different values of the suction or injection parameter S and the heat generation or absorption parameter d when M = d = R = Gr = A = B = 0 and Pr = 0.71.

S	D	Chamka	Acharya	Present study
0.45	0.5	0.82397	0.8225	0.8229
0.45	1	0.96191	0.9618	0.9619
0	0.5	0.94769	0.9462	0.9465
0	1	1.07996	1.0789	1.0791
-1.5	0.5	1.57077	1.5696	1.5696
-1.5	1	1.6603	1.6603	1.6603



Stagnation parameter L

Fig.1 shows that the effect of velocity profile with variation in velocity ratio parameter. We observed that the velocity profile increasing with increasing the values of L (L = $\frac{a}{b} = 0, 0.5, 1, 1.5, 2$). L is the ratio of free stream velocity (*ax*) to stretching velocity (*bx*) of the flow. Physically we observed that L < 1 when the stretching velocity of the surface of the flow exceeds the stream velocity of the flow. On similar manner these results are plotted for $L \le 1$ and L > 1.



Fig.2 and 3 shows that the effect of magnetic parameter M and suction parameter on velocity profile respectively. We fixed Pr = 0.71 and all other parameters to be zero in order to depict the effects of different parameters under investigation. From the fig.2 we observed that velocity profile increases when the values of magnetic parameter decreases due to Lorentz force. As the Lorentz force is resistive force which opposes the fluid motion. From fig 3. we observed that velocity is increases with decreases of suction parameter, because of the imposition of sheet suction the fluid brought is closure to the sheet and it reduces momentum boundary layer thickness.



Fig.4 shows that the influence of thermal radiation parameter on velocity profile . it is seen that as radiation parameter increases ,the velocity is increases. Fig. 5 and 6:shows that influence of Grashof number on velocity and temperature profile. We observed that velocity profile increase and temperature profile decrease

with Grashof number increase. It is evident from these points that the thermal boundary layer becomes thin for higher values of Grashof number and momentum boundary layer thickness increase with the increase in Grashof number. Fig7 shows that influence of Prandtl number on temperature, Pr increases the dimensionless temperature decreases. The thermal boundary layer thickness reduces with Pr, it is due to decrease of thermal diffusivity for the larger values of Prandtl value number.



Fig.8: The effect of thermal radiation parameter Fig.9: The effect of heat generation on on temperature profile. Temperature profile.

Fig 8. shows that effect of thermal radiation parameter on temperature profile, temperature profile increase with thermal radiation parameter increases. Fig.9 Shows that the influence of heat generation on temperature profile, temperature profile increase with increase in heat generation. The presence of a heat source in the boundary layer generates energy which causes the temperature of the fluid to increase. This increase in temperature produces an increase in the flow field due to the buoyancy effect.



Velocity profile.

Temperatur profile.

Fig.10. Shows that the influence of velocity slip parameter on velocity, velocity profile decrease with velocity slip parameter increase. Consequently, with an increment in the slip parameter the velocity boundary layer thickness decreases. This is due to the fact that as the velocity slip parameter increases in magnitude, allowing more fluid to slip past the sheet, the flow accelerates for a distance very close to the plate, whereas for a distance far away from the plate the opposite effect is observed. Fig 11. shows the effect of temperature with respect to thermal slip parameter B. From the graph we observe that the wall temperature $\theta(0)$ and thermal boundary layer thickness decreases when the values of slip parameter B increases.

V. Conclusion:

Investigation has been carried out numerically to study the effects of MHD stagnation point flow of a stretching sheet with Thermal radiation and Slip boundary conditions. The transformed nonlinear ordinary differential equations are solved by using Keller Box Method. The numerical results obtained agreed very well with previously published work and they are found to be in good agreement. The main observations of the present study are as follows:

1. The velocity of the fluid is decreases with an increase in both Magnetic parameter and Suction parameter.

2. Thermal boundary layer thickness decreases with an increase in both Grashof number and Prandtl number.

- 3. The velocity profile of the fluid is increases with increase in both Radiation parameter and Grashof number.
- 4. Thermal boundary layer thickness is increases with increase in Radiation parameter.
- 5. Temperature profile is decreases with an increase in Heat generation or absorption coefficient.
- 6. The velocity of the fluid is decreases with increase in velocity slip parameter.
- 7. The thermal boundary layer thickness is increases with an increases thermal slip parameter.

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