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Barotropic Bianchi Type Vi_0 Cosmological Model in General Relativity

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ABSTRACT:*Barotropic Bianchi Type VI_0 Cosmological Model in General Relativity is investigated and determinant solution is obtained by assuming $A = B^n$ and $p = \rho\gamma$. Physical and geometrical properties of the model are also discussed.*
KEY WORDS:*Bianchi type VI_0;Barotropic; Space time; Cosmology; Relativity.*

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I. Introduction

Cosmology is the branch of astronomywhich is deal with study of large structure of the universe with evolution. String cosmology model play significant role in cosmology i.e.study of the early stages of universe, galaxy formationand getting acceleration expansion phase of Universe. The Phase transitions in the early universe can give rise to microscopic topological defects such as vacuum domain walls, strings, walls bounded by strings, and monopoles connected by strings is initiated by Kibble and Vilenkin[1, 2]. The model of universe formed by massive string is initiated by Letelier, which was used as Bianchi type I and "Kantowski-Sachs" type of cosmological models [3, 4]. The basic virtues of inflation in the deflationary picture has been discussed by Gasperini*et al.*[5].

Bianchi type I-IX cosmological models are important in sense of strings, isotropic, homogeneous etc. In past five decades relativists has been interested in constructing string cosmological model. Borrow [6] has pointed out that Bianchi type VI_0 model of universe and He explained cosmological problem. Ruban [7] and Collins [8] discussed some exact solutions of Bianchi type VI_0 for perfect fluid distributions satisfying specific equation of state. Ellis and McCollum [9] investigated solution of Einstein field equation for Bianchi type VI_0 space time in stiff fluid. Dunn and Tupper [10] obtained the solution of a class of Bianchi type VI_0 perfect fluid cosmological model associated with electromagnetic field. Reddy and Rao [11] presented on some Bianchi type cosmological model in biometric theory of gravitation. Shri Ram [12] presented an algorithm for generating exact perfect fluid solution of Einstein field equation, not satisfying the equation of state, for spatially homogeneous cosmological model of Bianchi type VI_0. Singh and Singh [13] has been obtained the solution of string cosmological models with magnetic field in General Relativity. Some exact solution of string cosmological model has investigated by several researchers [14-17]. Xing-Xiang[18-20] has obtained solution of Bianchi string cosmological model with bulk viscosity and magnetic field.Bianchitype III for cloud string cosmological model described by Tikekar& Patel [21]. Chakraborty*et al.*[22, 23] investigated string cosmological model in general realtivity. In Bianchi Type VI_0 string cosmological model Tikekar and Patel [24] obtained some exact solutions. Bianchi Type I and Bianchi Type III investigated by Bali *et al.*[25-28]

Two parameter of Einstein's field equation is cosmological constant ∧ and gravitational constant G plays the role of coupling constant between geometry and matter in Einstein field equation. Shrimali and Joshi [29-32] obtained the solution of Bianchi type III cosmological model in general Relativity. Pradhan and Bali [33] obtained the solution of magnetized Bianchi type VI0 Barotropic massive string universe with decaying vacuum energy density. Verma and Ram [34] investigated the solution of Bianchi-Type VI0 Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants. Pradhanet. al.[35,36] obtained dark energy model in Bianchi Type VI0

Recently, Bali and Poonia [37] investigated Bianchi Type VI0 Inflationary Cosmological Model in General Relativity. Tyagi et.al [38-40]obtained Bianchi Type VI0 homogeneous cosmological model for anti-stiff perfect fluid for time dependent ∧ in general relativity Inhomogeneous cosmological model for stiff perfect fluid distribution in general relativity and Barotropic perfect fluid in creation field theory with time dependent cosmological model . Bali et .al [41] and Bhoyar et. al [42] has investigated Bianchi Type VI0 in general relativity.

II. Field Equation

We consider Bianchi type VI_0 space time metric in the form of

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2x} dy^2 + C^2 e^{2x} dz^2 \tag{1}$$

Where A, B and C are function of time t and m is constant. The energy momentum tensor for a bulk viscous fluid distribution is given by

$$T_i^j = (\rho + p) v_i v^j + p g_i^j \tag{2}$$

Here ρ, p is energy densities, pressure and string tension density respectively. The velocity vector of fluid satisfies

$$v_i v^j = -1 = -u_i u^j \tag{3}$$

$$u^i v_i = 0 \tag{4}$$

The vector $u_i u^j$ describes the direction of string or direction or anisotropy.

The Einstein field equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G T_{ij} \tag{5}$$

R_{ij} is known as Ricci tensor and T_{ij} is the energy momentum tensor for matter.

For the line element (1) and the field equation (5) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -8\pi G p \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -8\pi G p \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -8\pi G p \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = 8\pi G \rho \tag{9}$$

$$\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right) = 0 \tag{10}$$

Dot on B and C denotes the ordinary differentiation with respect to t.
Integration of equation (10) gives

$$B = LC \tag{11}$$

Where, L is constant of Integration. Without loss of generality we have to take $L = 1$. So that

$$B = C \tag{12}$$

The expression for scalar expansion, shear scalar, spatial volume is

$$\theta = v^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{13}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \tag{14}$$

$$S^3 = ABC \tag{15}$$

$$V = S^3 = ABC \tag{16}$$

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{17}$$

$$q = -1 + \frac{d}{dt}(H) \tag{18}$$

III. Solution Of Field Equation

We assumed that scalar expansion θ in this model is proportional to shear scalar σ

$$\theta \propto \sigma \tag{19}$$

So we have

$$A = B^n \tag{20}$$

Where n is any real number.
From equation (8)

$$(n+1)\frac{\ddot{B}}{B} + n^2\frac{\dot{B}^2}{B^2} - \frac{1}{B^{2n}} = -8\pi G p \tag{21}$$

From equation (8)

$$(2n+1)\frac{\dot{B}^2}{B^2} - \frac{1}{B^{2n}} = 8\pi G\rho \tag{22}$$

To get determinate solution we need extra condition. We assume

$$p = \rho\gamma \tag{23}$$

I.e. fluid is barotropic fluid.

$$\ddot{B} + \frac{(n^2 + (2\mathrm{n}+1)\gamma)}{(n+1)}\frac{\dot{B}^2}{B} = \left[\frac{1+\gamma}{n+1}\right]B^{1-2n} \tag{24}$$

Considering n=2

$$\frac{d}{dB}(\alpha^2\,\mathrm{B}^{2k_1}) = 2\,\mathrm{k}_2\,B^{2k_1-3} \tag{25}$$

Where $k_1 = \frac{4+5\gamma}{3}$ and $k_2 = \frac{1+\gamma}{3}$

$$B = \left[\left(\frac{2k_2}{k_1-1}\right)t + 2c\right]^{1/2} \tag{26}$$

$$B = \sqrt{k_3 t + C} \tag{27}$$

Where $k_3 = \frac{2(1+\gamma)}{1+5\gamma}$ and C is integrating constant.

Therefore equation (1) becomes

$$ds^2 = -dt^2 + \left(k_3 t + C\right)^{2n} dx^2 + \left(k_3 t + C\right)e^{-2x}dy^2 + \left(k_3 t + C\right)e^{2x}dz^2 \tag{28}$$

IV. Geometrical And Physical Parameter

From this model, we can find other geometrical and physical parameter. The expression for Hubble parameter H, Expansion scalar θ, Spatial volume V, Shear scalar σ^2 and Deceleration parameter are respectively given by

$$H = \frac{2}{3}\left(\frac{k_3}{k_3 t + C}\right) \tag{29}$$

$$\theta = \left(\frac{2k_3}{k_3 t + C}\right) \tag{30}$$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{k_3}{k_3 t + C}\right] \tag{31}$$

$$V = (k_3 t + C)^2 \tag{32}$$

$$q = -1 - \frac{k_3^2}{(\mathrm{k}_3\, t + C)^2} \tag{33}$$

$$\frac{\sigma}{\theta} = \frac{1}{2\sqrt{3}} \tag{34}$$

When T→0, The scalar expansion θ→∞ (infinite) and when T→∞, the scalar expansion θ→0 and declaration parameter is always negative therefore the model is decelerating. The spatial volume increases with time it become infinite for large value of T. the model is non-shearing and rotating as universe is expanding. Since $\lim_{T\to\infty} \frac{\sigma}{\theta}$ =constant, therefore anisotropy maintain for the all the time so for the large value of T model doesn't approach isotropy.

V. Conclusion

In this paper Barotropic Bianchi Type VI_0 Cosmological Model in General Relativity. We investigated that θ, σ, H is decrease with growth of cosmic time and V is increases with time. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ which gives us the anisotropy is maintained for all the time. The Model describes a continuously non-shearing, expanding, rotating.

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