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Amplification of Upper Hybrid Wave in Inhomogeneous Plasma Due to Nonlinear Force

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***Abstract**: A Generation Mechanism For Unstable Upper Hybrid Wave Is Considered In Inhomogeneous Plasma. The Resonant Mode Ion-Sound Wave Turbulence Supported By Inhomogeneous Plasma May Transfer Its Energy Through Resonant Interactions With Accelerated Electrons. These High Energetic Particles May Transfer Their Thermal Energy To Amplify Upper Hybrid Mode, Nonlinearly Through Modulated Field. Propagation Of Such An Unstable Upper Hybrid Wave May Be Responsible For Generation Of Unstable Upper Hybrid Wave Phenomena In Ionosphere. In This Paper We Have Estimated The Growth Rate Of Upper Hybrid Wave In Presence Of Ion-Sound Wave Turbulence Using Space Observational Data.*
***Key Words:** Wave Particle Interaction, Plasma Maser Effect, Inhomogeneous Plasma, Ion-Sound Wave Turbulence.*

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I. Introduction:

Space Plasma Is Basically Inhomogeneous. The Free Energy Of Ion-Sound Wave Turbulence Supported By An Inhomogeneous Media [1] Has A Role In Enhanced Radiations And Instabilities In Any Inhomogeneous Plasma System. Earlier, Vladimirov [2] Clarified The Role Of Nonlinear Wave-Particle Mechanism In Generation Of Instabilities. The Role Of Ion-Sound Wave Turbulent Field In Generating Instabilities In Inhomogeneous Plasma Through Resonant Interaction By A Non-Linear Force Was Recently Investigated By Borgohain And Deka [3]. The Amplification Of Resonant Waves At The Expense Of Resonant Mode Wave Energy Through Nonlinear Wave Particle Resonant Interaction Process Was First Proposed By Nambu [4] And Tsytovich [5] Independently For Homogeneous Plasma Model. In Reality, In Most Cases Confining Plasma Is Far From Homogeneous. Earlier, Some Authors Carried Out Mode Mode Coupling Process Through Wave Particle Interaction Process In Inhomogeneous Plasmas [6, 7].

In This Paper, We Are Considering Ion-Sound Wave Turbulence Field Supported By Inhomogeneous Plasma For Resonant Interaction With Plasma Particles. The Groups Of Plasma Particles Which Are Strongly In Phase Relation With Ion-Sound Wave Turbulent Field Are Accelerated [8]. If A Nonresonant Wave Is Present In The System We Have A Modulated Field With Component Of Resonant Mode Ion-Sound Wave Turbulence Field And Nonresonant Mode Of The System. Resonant Waves Are Those Which Satisfy Cherenkov Resonant Condition I.E. $\omega-\vec{k}.\vec{v}=0$. On The Other Hand Nonresonant Waves Are Those Which Do Not Satisfy Linear And Nonlinear Cherenkov Conditions I.E. $\Omega-\vec{K}.\vec{v}\neq 0$; And $\Omega-\omega-\left(\vec{K}-\vec{k}\right).\vec{v}\neq 0$. Here ω,Ω Are Frequencies Of Resonant And Nonresonant Waves And $\vec{k},\vec{K}$ Are The Corresponding Wave Numbers Respectively. The Accelerated Particles May Transfer Its Energy To Nonresonant Mode Through The Modulated Field And Nonresonant Mode May Be Amplified At The Expense Of Resonant Ion-Sound Wave Energy.

Here, We Estimated Growth Rate Of Upper Hybrid Wave Using Ionosperic Plasma Observational Data. Computational Result Shows That Upper Hybrid Wave May Grow At The Expense Of Low Frequency Ion-Sound Wave Turbulent Field Energy Through Nonlinear Force.

II. Mathematical Formulation:

We Consider Magnetized Plasma In Presence Of Ion-Sound Wave Turbulence. The Magnetic Field $B_0\left(x\right)$ With Negligible Gradient Is Taken Along The Z-Direction And The Spatial Gradient Of The System Is Taken In X-Direction. For This Purpose, The Particle Distribution Function [9] In The Case Of Inhomogeneous Plasma [10] Is Considered As

$$f_{j0}\left(\vec{v}, x-\frac{v_y}{\Omega_j}\right)=f_{j0}\left(v_\perp{}^2, v_\parallel, 0\right)\left\{1-\varepsilon'\left(x-\frac{v_y}{\Omega_j}\right)\right\} \tag{2.1}$$

Where $f_{j0}\left(v,0\right)$ Is The Distribution For The Guiding Centre; j Refers To Electron And Ion, $v_y = v_\perp \sin\theta$, $v_\parallel$ And $v_\perp$ Are The Component Of Velocity Along And Perpendicular To The Direction Of The External Magnetic Field And θ Is The Gyro Phase Of The Particle, $\Omega_j = \dfrac{e_j B_0}{mc}$ Is The Cyclotron Frequency And $\varepsilon' = \left[-\dfrac{1}{f_{j0}}\dfrac{\partial f_{j0}}{\partial x}\right]_{x=0}$ Is The Density Gradient.

The Geometry Of The Model Is Shown In Figure 1.

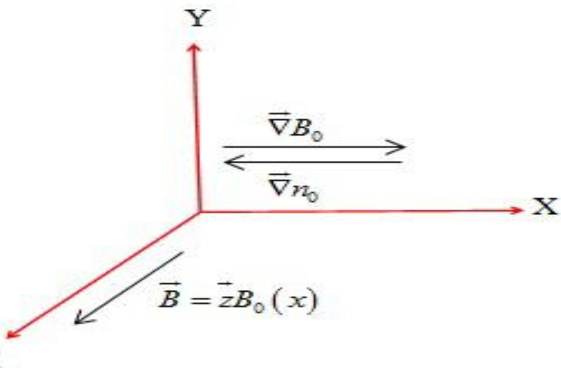

Figure 1: Geometrical Representation Of The Problem:

$\vec{K} = \left(K_\perp, 0, K_\parallel\right)$ Denotes The Upper Hybrid Wave Vector, $\vec{k} = \left(0, 0, k_\parallel\right)$ Denotes The Ion-Sound Wave Vector, $\nabla n_0\left(x\right)$ Is The Density Gradient Along Negative Of X-Direction, $B_0\left(x\right)$ Is The Magnetic Field Along Z-Direction.

The Interaction Of High Frequency Upper Hybrid Wave With Ion-Sound Wave Turbulence Is Governed By **Vlasov-Poisson Equations.**

$$\left[\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial \vec{r}}-\frac{e_j}{m_j}\left(\vec{E}+\frac{\vec{V}\times\vec{B}}{c}\right).\frac{\partial}{\partial \vec{v}}\right]F_{j0}\left(\vec{r},\vec{v},t\right)=0 \tag{2.2}$$

And

$$\vec{\nabla}.\vec{E} = -4\pi n_j e_j \int f_{j0}\left(\vec{r},\vec{v},t\right)d\vec{v}. \tag{2.3}$$

According To Linear Response Theory Of Turbulent Plasma [4], The Unperturbed Electron Distribution Function And Electric Field Are

$$F_{j0} = f_{j0} + \varepsilon f_{j1} + \varepsilon^2 f_{j2}, \tag{2.4}$$

And

$$\vec{E}_{0l} = \varepsilon \vec{E}_l \tag{2.5}$$

Where f_{j0} Is The Part Of Distribution Function And f_{j1} And f_{j2} Are The Fluctuating Part Due To The Low Frequency Ion-Sound Wave Turbulence And ε Is The Small Parameter Associated With Low Frequency Turbulence.

To The Order Of ε ,We Obtain From Equation (2.2)

$$\left[\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial \vec{r}}-\frac{e_j}{m_j}\left(\vec{E}+\frac{\vec{v}\times B_0\left(x\right)}{c}\right).\frac{\partial}{\partial \vec{v}}\right]f_{j1}\left(t,\vec{r},\vec{v}\right)=\frac{e_j}{m_j}\vec{E_l}.\frac{\partial}{\partial \vec{v}}f_{j0}.$$

(2.6)

We Consider Ion-Sound Wave Propagation Vector As $\vec{k}=\left(0,0,k_{\parallel}\right)$ And The Turbulence Field As $\vec{E}_l=\left(0,0,\vec{E}_{l\parallel}\right)$.

To Find f_{j1}, We Use Fourier Transform Of The Form

$$A\left(\vec{r},\vec{v},t\right)=\sum_{\vec{k},\omega}A\left(\vec{k},\vec{v},\omega\right)\exp\left[i\left(\vec{k}.\vec{r}-\omega t\right)\right] \tag{2.7}$$

The Fluctuating Parts Of The Low Frequency Turbulent Field f_{j1} Is Now Obtained From Equation (2.6)

$$f_{j1}=\int_{-\infty}^{0}\left(\frac{e}{m_j}\vec{E}_l.\frac{\partial}{\partial\vec{v}}f_{j0}\right)\exp\left[i\left\{\vec{k}.\left(\vec{r}'-\vec{r}\right)-\omega t\right\}\right]d\tau=\frac{ie}{m_j}\left(\frac{E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}f_{j0}\left(\vec{v}\right)}{\omega-k_{\parallel}v_{\parallel}+i0^{+}}\right),$$

(2.8)

$i0^{+}$ Is The Small Imaginary Part Of ω.

We Now Perturb The Quasisteady State By The Test Of Upper Hybrid Wave Field $\mu\delta\overrightarrow{E_h}$ With Wave Vector $\vec{K}=\left(K_{\perp},0,K_{\parallel}\right)$, Electric Field $\left(\delta E_{hx},0,\delta E_{hz}\right)$, Magnetic Field $\delta\vec{B}=\left(0,0,0\right)$. Thus Total Perturbed Electric Field And Magnetic Field And The Electron Distribution Function Due To This Perturbation Are

$$\delta\vec{E}=\mu\delta\overrightarrow{E_h}+\mu\varepsilon\delta\overrightarrow{E_{lh}}+\mu\varepsilon^2\Delta\vec{E}, \tag{2.9}$$

$$\delta\vec{B}=0, \tag{2.10}$$

$$\delta f_j=\mu\delta f_h+\mu\varepsilon\delta f_{lh}+\mu\varepsilon^2\Delta f; \tag{2.11}$$

Where $\delta\overrightarrow{E_{lh}}$ And $\delta\vec{E}$ Are The Modulation Field, δf_h Is The Fluctuating Part Of Distribution Function Due To High Frequency Upper Hybrid Wave And δf_{lh} And Δf Are Particle Distribution Function Corresponding To Modulated Fields.

Substituting These Values In Vlasov Equation (2.6) For The Perturbed State Conditions

$$\left[\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial\vec{r}}-\frac{e}{m_j}\left\{\left(\overrightarrow{E_{l0}}+\delta\vec{E}\right)+\frac{\vec{v}\times B_0\left(x\right)}{c}\right\}.\frac{\partial}{\partial\vec{v}}\right]\left(F_{j0}+\delta f_j\right)=0$$

$$\Rightarrow\left[\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial\vec{r}}-\frac{e}{m_j}\left\{\left(\varepsilon\overrightarrow{E_l}+\mu\delta\overrightarrow{E_h}+\mu\varepsilon\delta\overrightarrow{E_{lh}}+\mu\varepsilon^2\Delta\vec{E}\right)+\frac{\vec{v}\times B_0\left(x\right)}{c}\right\}.\frac{\partial}{\partial\vec{v}}\right]$$
$$\left(f_{j0}+\varepsilon f_{j1}+\varepsilon^2 f_{j2}+\mu\delta f_h+\mu\varepsilon\delta f_{lh}+\mu\varepsilon^2\Delta f\right)=0$$

(2.12)

To The Order Of μ, $\mu\varepsilon$ And $\mu\varepsilon^2$, We Get

$$P\delta f_h=\frac{e}{m_j}\delta\overrightarrow{E_h}.\frac{\partial}{\partial\vec{v}}f_{j0}, \tag{2.13}$$

$$P\delta f_{lh}=\frac{e}{m_j}\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}f_{j0}+\frac{e}{m_j}\delta\overrightarrow{E_h}.\frac{\partial}{\partial\vec{v}}f_{j1}+\frac{e}{m_j}\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_h$$

(2.14)

$$P\Delta f=\frac{e}{m_j}\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_{lh}+\frac{e}{m_j}\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}f_{j1} \tag{2.15}$$

Where The Operator P Is Given By

$$P=\frac{\partial}{\partial t}+\vec{v}.\frac{\partial}{\partial\vec{r}}-\frac{e}{m_j}\left\{\frac{\vec{v}\times B_0\left(x\right)}{c}\right\}.\frac{\partial}{\partial\vec{v}}\quad . \tag{2.16}$$

To Obtain The Fluctuating Part δf_h Of The Distribution Due To The High Frequency Upper Hybrid Wave, We Integrate Equation (2.13) Over The Particle Trajectories By Using Fourier Transforms And The Method Of Characteristics [9]

$$\delta f_h\left(\vec{K},\Omega\right)=\frac{e}{m_j}\int_{-\infty}^{0}\left(\delta\overrightarrow{E_h}.\frac{\partial}{\partial\vec{v}}f_{j0}\right)\exp\left[i\left\{\vec{K}.\left(\vec{r}^{'}-\vec{r}\right)-\Omega\left(t^{'}-t\right)\right\}\right]dt^{'}$$

$$=\frac{e}{m_j}\left(-i\delta E_h\right)\frac{\dfrac{\partial f_{j0}}{\partial v_{\parallel}}}{K_{\parallel}v_{\parallel}-\Omega} \tag{2.17}$$

Similarly Fluctuating Parts δf_{lh} Is Obtain As

$$\delta f_{lh}\left(\vec{K},\Omega\right)=\frac{e}{m_j}\int_{-\infty}^{0}\left(\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}f_{j0}+\delta\overrightarrow{E_h}.\frac{\partial}{\partial\vec{v}}f_{j1}+\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_h\right)\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=I_1+I_2+I_3 \tag{2.18}$$

Where

$$I_1=\frac{e}{m_j}\int_{-\infty}^{0}\left(\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_h\right)\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=\left(\frac{e}{m_j}\right)^2\int_{-\infty}^{0}E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}\left(\left(-i\delta E_h\right)\frac{\dfrac{\partial f_{j0}}{\partial v_{\parallel}}}{K_{\parallel}v_{\parallel}-\Omega}\right)\times\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=-\left(\frac{e}{m_j}\right)^2E_{l\parallel}\delta E_h\frac{\partial}{\partial v_{\parallel}}\left(\frac{\dfrac{\partial f_{j0}}{\partial v_{\parallel}}}{K_{\parallel}v_{\parallel}-\Omega}\right)R_{a,b} \tag{2.19}$$

$$I_2=\frac{e}{m_j}\int_{-\infty}^{0}\left(\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}\delta f_{j0}\right)\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=\frac{e}{m_j}\int_{-\infty}^{0}\left(\delta E_{lh\perp}\frac{\partial}{\partial v_{\perp}}+\delta E_{lh\parallel}\right)f_{j0}\left(v_{\perp}^{2},v_{\parallel}\right)\times\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=\frac{e}{m_j}\int_{-\infty}^{0}\left[\frac{\delta E_{lh}K_{\perp}}{\left|K-k\right|}\left(-\frac{m_j}{T_j}v_{\perp}-\frac{\varepsilon^{'}}{\Omega_j}\right)f_{j0}\left(\vec{v}\right)+\frac{\delta E_{lh}\left(K_{\parallel}-k_{\parallel}\right)}{\left|K-k\right|}f_{j0}\left(\vec{v}\right)\right]\times$$

$$\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}^{'}-\vec{r}\right)-\left(\Omega-\omega\right)\tau\right\}\right]d\tau$$

$$=\frac{ie}{m_j}\frac{\delta E_{lh}}{\left|K-k\right|}\left[\frac{m_j}{T_j}\left\{1+\left(\Omega-\omega-\frac{\varepsilon^{'}K_{\perp}T_j}{m_j\Omega_j}\right)R_{a,b}\right\}-\left(K_{\parallel}-k_{\parallel}\right)R_{a,b}\right]\times f_{j0}\left(\vec{v}\right)$$

(2.20)

$$I_3 = \frac{e}{m_j}\int_{-\infty}^{0}\left(\delta\overrightarrow{E_h}.\frac{\partial}{\partial\vec{v}}\delta f_{j1}\right)\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}'-\vec{r}\right)-(\Omega-\omega)\tau\right\}\right]d\tau$$

$$= \frac{e}{m_j}\int_{-\infty}^{0}\delta E_{hz}\frac{\partial}{\partial v_{\parallel}}\left\{\frac{ie}{m_j}\frac{E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}f_{j0}}{\omega-k_{\parallel}v_{\parallel}+i0^+}\right\}\exp\left[i\left\{\left(\vec{K}-\vec{k}\right).\left(\vec{r}'-\vec{r}\right)-(\Omega-\omega)\tau\right\}\right]d\tau$$

$$= \left(\frac{e}{m_j}\right)^2\delta E_h\frac{\partial}{\partial v_{\parallel}}\left\{\frac{E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}f_{j0}}{\omega-k_{\parallel}v_{\parallel}+i0^+}\right\}R_{a,b} \tag{2.21}$$

III. Nonlinear Force:

We Obtain From Equation (2.15)

$$\Delta f = \int_{-\infty}^{0}F\left(\vec{K},\Omega\right)\exp\left[i\left\{\vec{K}.\left(\vec{r}'-\vec{r}\right)-\Omega\tau\right\}\right]d\tau$$

$$= \frac{F\left(\vec{K},\Omega\right)}{i\left(K_{\parallel}v_{\parallel}-\Omega+a\Omega_j\right)}\sum_{p,q}J_p(\alpha)J_q(\alpha)\exp\left\{i(q-p)\theta\right\}$$

$$= \frac{F\left(\vec{K},\Omega\right)}{i}S_{p,q} \tag{3.1}$$

Where $F\left(\vec{K},\Omega\right) = \frac{e}{m_j}\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_{lh} + \frac{e}{m_j}\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}\delta f_{j1}$.

Now The Nonlinear Force F_N Acting On Unit Volume Of Particles Can Be Written As

$$F_N = n_j\frac{\partial}{\partial t}\int m_j v\Delta f dv = n_j\frac{\partial}{\partial t}\int m_j v\Delta f\exp\left[i\left\{\vec{K}.\vec{r}-\Omega\tau\right\}\right]dv,$$

Where n_j Being The Number Of Ion Density.

$$F_N = m_j n_j\int\frac{F\left(\vec{K},\Omega\right)}{i\left(K_{\parallel}v_{\parallel}-\Omega+a\Omega_j\right)}(-i\Omega)\sum_{p,q}J_p(\alpha)J_q(\alpha)\exp\left\{i(q-p)\theta\right\}\times$$

$$\exp\left[i\left\{\vec{K}.\vec{r}-\Omega\tau\right\}\right]vdv$$

$$= -m_j n_j\Omega\int\left(\frac{e}{m_j}\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_{lh} + \frac{e}{m_j}\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}\delta f_{j1}\right)J_{p,q}vdv.$$

The Z-Component Of This Nonlinear Force Are

$$F_{NZ1} = -m_j n_j\int\Omega\left(\frac{e}{m_j}\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_{lh}\right)v_{\parallel}dvJ_{p,q} = -en_j\Omega\int\left(\overrightarrow{E_l}.\frac{\partial}{\partial\vec{v}}\delta f_{lh}\right)v_{\parallel}dvJ_{p,q}$$

(3.2)

And

$$F_{NZ2} = -m_j n_j\Omega\int\left(\frac{e}{m_j}\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}f_{j1}\right)v_{\parallel}dvJ_{p,q} = -en_j\Omega\int\left(\delta\overrightarrow{E_{lh}}.\frac{\partial}{\partial\vec{v}}f_{j1}\right)v_{\parallel}dvJ_{p,q}.$$

To Evaluate The Nonlinear Force, We Consider Resonant Interaction Condition $\omega-\vec{k}.\vec{v}=0$ Of Ion-Sound Wave Turbulence. Here We Consider Two Situations, First $K_{\parallel}v_{\parallel}>\Omega$ And Secondly, The Gyrophase θ To Be

Very Small. We Also Consider The Fact That The Most Dominant Contribution Comes From Bessel's Terms Where $a = b = p = q = 0$.

Using The Expressions For I_1, I_2 And I_3 From Equations (2.19),(2.20) And (2.21) Respectively, We Obtain Expression For δf_{lh} In Eq.(2.18). After Lengthy Calculations, We Get F_{NZ1} From Eq.(3.2), Where We Retain Only Dominant Terms As

$$F_{NZ1} = 2\sqrt{\pi} e n_j \Omega^2 \delta E_h \left(\frac{e}{m_j}\right)^2 \frac{1}{|k| K_{\parallel}} \frac{1}{v_e v_d^{\,2}} E_{l\parallel} \left(E_{l\parallel} \frac{\varepsilon'}{\Omega_e} + E_{l\parallel} \frac{v_d}{v_e^{\,2}} \right) \exp\left\{ -\left(\frac{v_d}{v_e}\right)^2 \right\}$$

(3.3)

Where $v_d = \dfrac{\omega}{k_{\parallel}}$ Is The Ion Sound Velocity. In The Above, We Have Considered The Imaginary Parts Because They Gives To The Growth Of Plasma Waves Through Plasma Interaction For Which The Condition $\omega - \vec{k}.\vec{v} = 0$. Here, We Have Used

$$\mathrm{Im}\left(\frac{1}{-\omega + kv + i0^+}\right) = -i\pi\delta(\omega - kv) \text{ And } \int_{-\infty}^{\infty} \delta(\omega - kv) G(v)\, dv = \frac{1}{|k|} G\left(\frac{\omega}{k}\right).$$

Here, We Are Not Considering The Real Part In The 3evaluation Process, As The Real Part Will Give The Nonlinear Frequency Which Is Neglected In Weak Turbulence Approximation [4]. Now The Second Part Of Nonlinear Force F_{NZ2}, As

$$F_{NZ2} = -e n_j \Omega \int \left(\delta \overrightarrow{E_{lh}} . \frac{\partial}{\partial \vec{v}} f_{j1} \right) v_{\parallel} dv J_{p,q}$$

Comes From Of Polarization Coupling Term Whereas F_{NZ1} Comes From Contributions Of Direct Coupling Term. We Know That The Direct Coupling Term Dominates Over The Polarization Coupling Term For Growth Rate Of Upper Hybrid Wave, So We Only Consider F_{NZ1} In Our Discussion. Therefore,

$$F_{Nz} \cong F_{NZ1} \cong 2\sqrt{\pi} e n_j \Omega^2 \delta E_h \left(\frac{e}{m_j}\right)^2 \frac{1}{|k| K_{\parallel}} \frac{1}{v_e v_d^{\,2}} E_{l\parallel} \left(E_{l\parallel} \frac{\varepsilon'}{\Omega_e} + E_{l\parallel} \frac{v_d}{v_e^{\,2}} \right) \exp\left\{ -\left(\frac{v_d}{v_e}\right)^2 \right\}$$

(3.4)

IV. Nonlinear Dispersion Relation:

We Use The Electron Fluid Equation With An Additional Term Of Nonlinear Force [11] F_N Given By Chen [12] To Get Dispersion Relation Of Upper Hybrid Wave In Presence Of Ion-Sound Wave Turbulence.

$$Mn\left(\frac{\partial v_j}{\partial t} + v_j.\nabla v_j\right) = en\delta E_h - \nabla p + F_N \tag{4.1}$$

M Is The Mass Of Electron. We Use Plasma Approximation $n_j = n$. Now Assume $\delta E_h = \nabla\phi$, And Using The Electron Fluid Equation Of State To Get $\nabla p = \gamma_j k_B T_j \nabla n$, We Have From Eq.(4.1)

$$Mn\left(\frac{\partial v_j}{\partial t} + v_j.\nabla v_j\right) = en\delta E_h - \gamma_j k_B T_j \nabla n + F_N \tag{4.2}$$

Where k_B Is The Boltzmann Constant. Using Fourier Transformations In Eq. (4.2) And Assuming Steady State $n = n_0$ And $v_{j0} = 0$, We Get

$$-iMn_0\Omega v_{j1} = -ien_0 K\phi_1 - i\gamma_j k_B T_j \delta n_h K + F_N ,$$

(4.3)

Where δn_h Is The Density Fluctuations, And v_{j1} And ϕ_1 Are The Velocity Fluctuation And Potential Fluctuation Respectively. Taking The Z-Component,

$$-iMn_0\Omega v_{j1z} = -ien_0K\phi_1 - i\gamma_j k_B T_j \delta n_h K + F_{Nz} \quad .$$

(4.4)

We Consider The Mass Of Electron $m \to 0$, And Therefore From Boltzmann Relation For Electrons

$$n_j = n_0 \exp\left(\frac{e\phi_1}{k_B T_j}\right) = n_0\left(1 + \frac{e\phi_1}{k_B T_j} + ...\right),$$

(4.5)

Neglecting The Higher Order Terms, We Get The Perturbation In Density Of Electrons, Therefore Density Of Electron Is

$$\delta n_h = n_0 \frac{e\phi_1}{k_B T_j}. \tag{4.6}$$

Now Consider Equation Of Continuity

$$\frac{\partial n}{\partial t} + \nabla.\left(nv_j\right) = 0 \tag{4.7}$$

Using Fourier Transforms In Eq. (4.7) In Steady State, We Get

$$-i\Omega\delta n_h + in_0 K.v_{j1z} = 0. \tag{4.8}$$

Now Using Eqs. (4.4) And (4.6) In Eq. (4.8), We Get Dispersion Relation Of Upper Hybrid Wave, Due To Direct Coupling Term In Presence Of Ion-Sound Wave Turbulence As

$$\Omega^2 = K^2\left(\frac{k_B T_j + \gamma_j k_B T_j}{M}\right) + i\frac{K}{M}.\frac{F_{Nz}}{\delta n_h}. \tag{4.9}$$

V. Growth Rate:

Now We Put $\Omega = \Omega_R + i\gamma$, Where Ω_R And γ Are The Real Frequency And Growth Rate Of Upper Hybrid Wave. Neglecting Nonlinear Terms In Eq.(4.9), We Get

$$\Omega_R = |K|\left(\frac{k_B T_j + \gamma_j k_B T_j}{M}\right)^{\frac{1}{2}} = |K|V \tag{5.1}$$

Where V Is The Phase Velocity Of Upper Hybrid Wave, And

$$\gamma = \frac{K}{2M\Omega_R}\frac{F_{Nz}}{\delta n_h} \tag{5.2}$$

Using Eq.(3.4) And (5.1) In Eq.(5.2), We Get

$$\frac{\gamma}{\Omega} = \frac{\sqrt{\pi}}{M}\Omega\left(\frac{e}{m_j}\right)^2 \frac{1}{|k_{\Box}|K_{\Box}} \frac{k_B T_j}{v_e v_d{}^4 V} E_{l\Box}\left(E_{l\Box}\frac{\varepsilon'}{\Omega_e} + E_{l\Box}\frac{v_d}{v_e{}^2}\right)\exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}$$

(5.3)

Which Gives The Growth Rate Of Upper Hybrid Wave Direct Interaction With Ion-Sound Wave Turbulence In Inhomogeneous Plasma. In The Region With Weak Density Gradient, Consider $\varepsilon' \Box 0$ In Eq.(5.3), We Get The Growth Rate For Such Regions As

$$\frac{\gamma}{\Omega} = \frac{\sqrt{\pi}}{M}\Omega\left(\frac{e}{m_j}\right)^2 \frac{1}{|k_{\Box}|K_{\Box}} \frac{k_B T_j}{v_e{}^3 v_d{}^3 V} E_{l\Box}{}^2 \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\} \tag{5.4}$$

For Laboratory Plasma T_j Is A Common Occurrence [13], And Then We Have $V = \left(\frac{k_B T_j}{M}\right)^{\frac{1}{2}}$. Therefore Our Growth Rate From Eq. (5.2) As

$$\frac{\gamma}{\Omega} = \sqrt{\pi}\Omega\left(\frac{e}{m_j}\right)^2 \frac{1}{|k_{\Box}|K_{\Box}} \frac{V}{v_e v_d{}^4} E_{l\Box}\left(E_{l\Box}\frac{\varepsilon'}{\Omega_e} + E_{l\Box}\frac{v_d}{v_e{}^2}\right)\exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}$$

(5.5)

VI. Discussion

In Our Investigation We Are Considering The Instability Of High Frequency Plasma Wave In Presence Of Low Frequency Plasma Wave Through Plasma Wave Particle Interaction Process Considering Nonlinear Force Involved In It. The Growth Rate Of Upper Hybrid Wave In Presence Of Ion-Sound Wave Turbulence Has Been Estimated From The Consideration Of Nonlinear Force. We Observe That The Growth Rate Is Directly Related To The Nonlinear Force F_{Nz}. Therefore We Conclude That Nonlinear Force F_N Has A Role In The Amplification Process Of Electrostatic Wave In Inhomogeneous Plasmas. Therefore The Nonlinear Force F_N Has The Important Role In The Instability In Inhomogeneous Plasmas. Also Unstable Upper Hybrid Wave Is Continuously Connected With Electromagnetic Radiations While Propagating Through Inhomogeneous Plasma Medium [14].
It Also Observed That In An Inhomogeneous Plasma, Ion-Sound Wave Turbulence Has A Common Feature And Its Turbulence Wave Energy Is Available In The Parallel Direction Of The Confining Field. Therefore Neglecting The Power Of v_e Greater Than One And v_d Greater Than Four, We Get From Eq. (5.5)

$$\frac{\gamma}{\Omega} = \sqrt{\pi}\,\frac{\Omega}{\Omega_e}\left(\frac{e}{m_j}\right)^2 \frac{1}{|k_\parallel| K_\parallel}\frac{V}{v_e v_d^{\,4}} E_{l\parallel}^{\,2}\varepsilon' \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\} \quad .$$

(6.1)

To Get An Estimation Of Growth Rate Of Unstable Upper Hybrid Wave We Use Ionospheric Plasma Data [15, 16, 17].

The Plasma Parameters And Upper Hybrid Parameters

$$\Omega \approx \Omega_e \approx 10^6\,Hz,\ v_e \approx 4.19\times10^9\,ms^{-1},\ K_\parallel \approx 10^{-3}\,m^{-1}, \frac{e}{m_j} \approx 1.75\times10^{11}\,Ckg^{-1}.$$

The Ion-Sound Wave Parameters In Space

$$k_\parallel \approx 10^{-5}\,m^{-1},\ v_d \approx 10^6\,ms^{-1}.$$

The Electrostatic Plasma Wave Electric Field Intensity

$$E_{l\parallel} \approx 10^{-4}\,Vm^{-1}.$$

With These Data The Estimated Growth Rate From Polarization Coupling Term From Eq.(5.5) Is Found As,

$$\frac{\gamma}{\Omega} \approx 10^{-8}\varepsilon' \qquad (6.2)$$

Using Observational Value [18] Of Density Gradient Scale Length In Magnetosphere Near The Auroral Region And Considering Its Associated Relation With ε', We Put $\varepsilon' = 1$ In (6.2), To Get The Growth Rates For Upper Hybrid Wave In Inhomogeneous Plasma From Polarization Coupling Term As

$$\frac{\gamma}{\Omega} = 10^{-8} \qquad (6.3)$$

In View Of The Fact [14] That Unstable Upper Hybrid Wave Is Continuously Connected Electromagnetic Radiation While Propagating Through Inhomogeneous Medium, The Generation Of Auroral Kilometric Radiation May Be Associated With Unstable Upper Hybrid Wave In Presence Of Ion-Sound Wave Turbulence.
We Have Plotted In Figure2, The Growth Rate For Upper Hybrid Wave With Different Values Of The Density Gradient Parameter ε'.

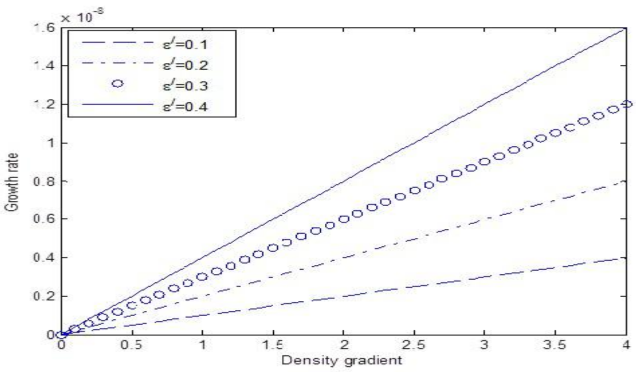

Figure 2: The Growth Rate Of Upper Hybrid Wave: Density Gradient Of Plasma Regions Influences The Growth

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