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Significance Of Stress Intensity Factor On Failure Behaviour Of Reinforced Concrete Beam (A case study of: Mode I Fracture)

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Abstract:*The term Stress Intensity Factor – SIF, is one of the fracture parameters which describes stress intensity around crack tip, represented by variable K. Depends on its critical value K_c, structural failure capacity might be analyzed based on the view of point of fracture mechanics. This study focuses on how reinforcement influences the rate of crack propagation. Wedge forces developed by cohesiveness between re-bar and concrete is the main concern in transforming brittle failure to plastic failure by means reducing the value of K. Specimen with three point mode I fracture reinforced high strength concrete beam was used in analytical processing. It is a mode I fracture beam of (150x300) mm with 100 mm initial crack. This specimen is reinforced by 4#12mm steel bar. Under wedge forces due to a center bending load, led a value of K equal to 482.70 $Nmm^{-1.5}$ for beam without reinforcement, whereas 441.613 $Nmm^{-1.5}$ for beam with reinforcement.Superposition of these both actions prevails the net of stress intensity factor $K = 41.087Nmm^{-1.5}$. By applying the term strain energy released rate G in conjunction with stress intensity factor K through the relationship $K = \sqrt{EG}$, prevails, netto of G = 35 N/m , a value under which normal plain concrete would fail.*

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I. Background

Concrete structures are full of cracks[1] . This phenomenon comes from initial internal discontinuity which occurs due to some reasons, such as imperfection during the mixing of constituent component of concrete material. Internal cracks, have the potential to increase the stress intensity factor, K, around the crack-tip. K is known as fracture toughness supplied by energy absorption due to external loads. The magnitude of K is significantly influenced by the crack tip geometry and external load configuration.

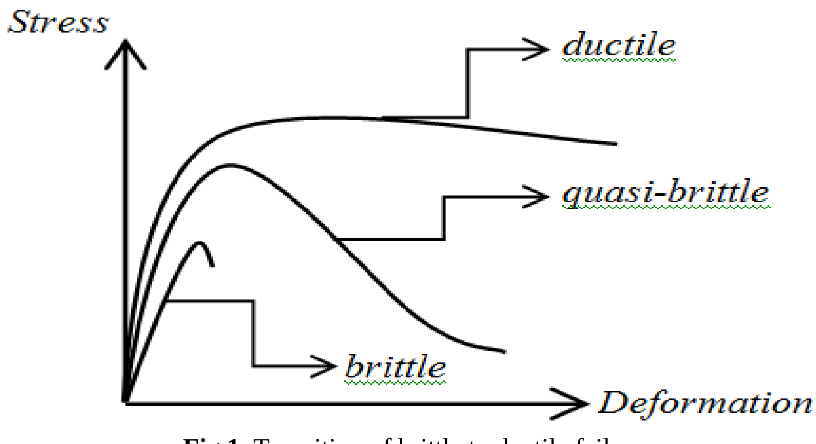

Fig 1. Transition of brittle to ductile failure

From the standpoint of concrete fracture mechanics, the accumulated stress intensity at the crack tip should be limited such that, under the increment of loading, stable growth of large fracture process zone (FPZ) would be in charge before the ultimate load is reached. This is what the concept of ductility underlies especially in earthquake resistant structures. By adding reinforcement, Reinforcement is a driving force that generates composite action between re-bars and concrete, hence, it is able to divert the brittle failure gradually to quasi-brittle, and finally to ductile, as shown in Fig.1

II. Literature Review

2.1 Stress Intensity Factor

The stress intensity factor K was developed in 1957 by Irwin[5], defines the magnitude of the local stresses around the crack tip. Please note that this factor depends on loading, crack size, crack shape, and geometric boundaries, with the general form given by

$K = \sigma\sqrt{\pi a}$……...for infinite structure 1

$K = \sigma\sqrt{\pi a}(g_1)\left(\frac{a}{b}\right)$…..for finite structures 2

In case of bending (shown by Fig.2), eq.2 is applied where a is the length of initial crack, b and t are the beam dimensions, and g_1 is the geometry function, calculated as

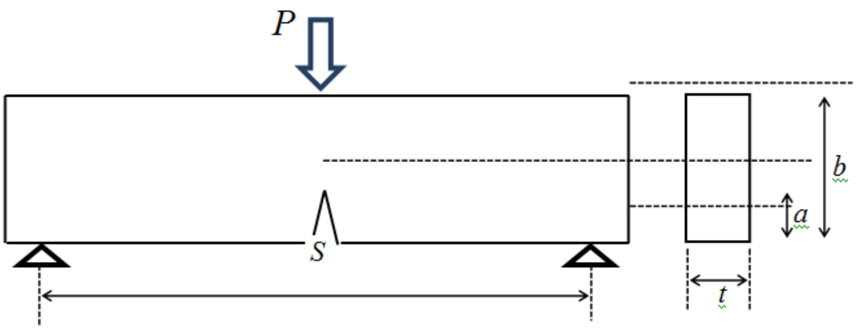

Fig.2 Three point bending test

$$g_1\left(\frac{a}{b}\right) = \frac{1.0 - 2.5\,a/b + 4.49(a/b)^2 - 3.98(a/b)^3 + 1.33(a/b)^4}{\left(1 - {}^{a}/_{b}\right)^{3/2}} \quad 3$$

The term stress σ is the stress due to P, acts perpendicular the crack face, counted as

$$\sigma = \frac{1.5PS}{2td^2} \quad 4$$

On the concept of strength state design, capacity of structure is modeled on the internal forces equilibrium (concrete compressive and steel tensile). For bending the following equation must be met,

$$M_{nf} = M_u \quad 5$$

where M_{nf} is nominal moment which must be greater at least equal to the ultimate external moment. That postulation might be expressed by the compatibility between the energy absorbed when loading and energy released when unloading. Please be noticed that in case of cracks, unloading occurs simultaneously with crack propagation for quasi brittle materials (analog with yielding for ductile materials). Griffith (1920)[4] proposed the concept of strain energy release rate to demonstrate the equilibrium states of a structure with a crack as

$$\Pi = U - F + W \quad 6$$

where Π is the total potential energy, U is the strain energy, F is the work done by the applied force, and W is the energy for crack formation. The necessary condition for the structure in equilibrium states is that the first order derivative of Π is equal to zero during an infinitesimal crack extension da, that is

$$\frac{\partial}{\partial a}(U - F + W) = 0 \quad 7$$

or

$$\frac{\partial}{\partial a}(F - U) = \frac{\partial W}{\partial a} \quad 8$$

Applied to mode I fracture beam (Fig.2) of quasi-brittle materials such as concrete, eq. 8 may be re-written as[8]

$$G_q = G_c + G_\sigma \quad 9$$

G_q is mode I strain energy release rate for quasi-brittle materials, represented by $\frac{\partial F}{\partial a}$, G_c is critical strain energy release rate for linear elastic materials, represented by $\frac{\partial W}{\partial a}$ and G_σ is strain energy release rate due to closing pressure represented by $\frac{\partial U}{\partial a}$. For linear brittle materials, where closing pressure (due to bridging effect between aggregate and matrix) is not dominant, eq.9 might be written as

$$G = G_c \quad 10$$

meaning energy needed to create one unit crack area. G_c may be re-written in the form[3,7]

$$G_c = \frac{\sigma_f^2 \pi a}{E} \quad 11$$

where σ_f is the failure stress, E is modulus of elasticity, and a is the length of initial crack. By substituting the critical value of eq.1, resulting in

$$G_c = \frac{K_c^2}{E} \, 12$$

More generally, noncritical values of G and K are related in the same way.

2.2 Wedge Forces of Single Crack Element

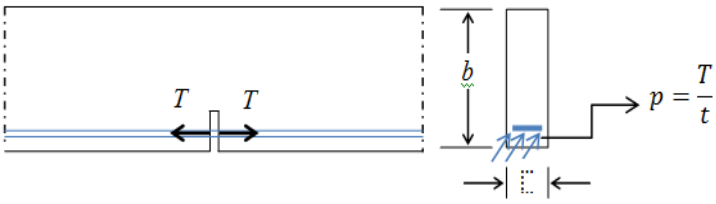

Fig.3 Wedge forces of a single crack reinforced beam

Viewing from fracture mechanics concepts, reinforcement has to be designed not only to take over tensile stresses, but also to control the opening of main crack by contributing wedge forces $p = \frac{T}{t}$ as shown in Fig.3. This means that wedge forces may eliminate the stress intensity factor at the crack tip which can be calculated using the Green function as [2,9]

$$K_I = \frac{2p}{\sqrt{\pi a}} g_1\left(\frac{a}{b}, \frac{x}{a}\right) \qquad 13$$

where

$$g_1\left(\frac{a}{b}, \frac{x}{a}\right) = \frac{3.52\left(1-\frac{x}{a}\right)}{\left(1-\frac{a}{b}\right)^{\frac{3}{2}}} - \frac{4.35-5.28\frac{x}{a}}{\left(1-\frac{a}{b}\right)^{\frac{1}{2}}} + \left[\frac{1.30-0.30\left(\frac{x}{a}\right)^{\frac{1}{2}}}{\sqrt{1-\left(\frac{x}{a}\right)^2}} + 0.83 - 1.76\frac{x}{a}\right]\left[1-\left(\frac{x}{a}\right)\frac{a}{b}\right] \quad 14$$

III. Case Study

A reinforced concrete beam of 150mm x 300mm dimension with span S = 1000mm shown below, subjected to a centralized load P at the middle span. Beam with a single crack is made of material with properties given as follows[3]

a. Compressive strength $f_c' = 104.30\ MPa$
b. Fracture toughness $K_{1c} = 22.136\ Nm^{-\frac{3}{2}}$
c. Modulus of elasticity $E = 48000\ MPa$
d. Yield strength of steel bar $f_{ys} = 590\ MPa$

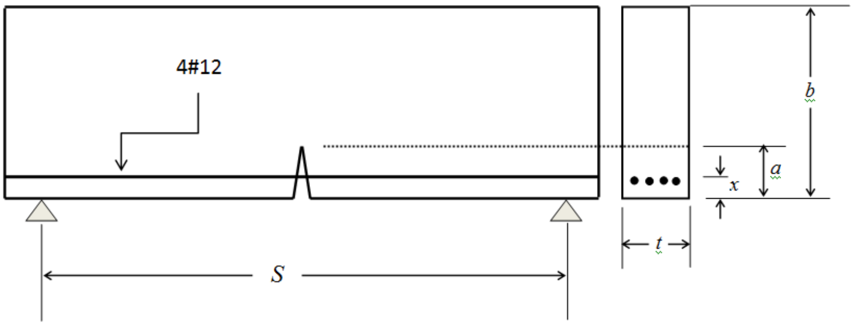

Fig. 4 Three point bending beam with tensile opening

If the initial crack a = 100mm, with 4#12mm reinforcement which met the minimum steel ratios in reinforced concrete beams[8]. This study investigated whether the reinforcement capacity of high strength concrete beam is sufficient to reduce the intensity of the stress at the crack tip so that the brittle failure can be diverted to quasi-brittle failure.Consider the following strain and stress diagram shown by Fig.5

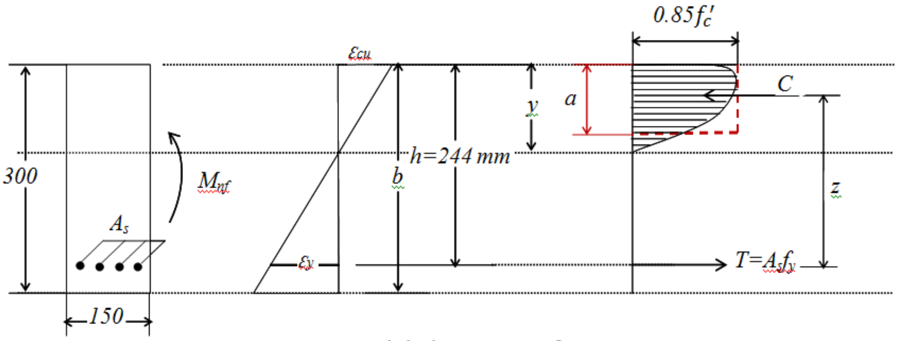

Fig.5 Beam with balancing reinforcement

1 Determination of the location of neutral axis (y)

Neutral axis is determined based on the linear relationship shown by Fig 5. If the yield strain of steel bar $\varepsilon_y = 0.0028$ and the ultimate strain of concrete $\varepsilon_{cu} = 0.002$ prevails y = 101.67 mm.

2 Determination of moment capacity M_{nf}

$$C = 0.85 f_c' \text{ x } 150 \text{ x a}$$

$$T = fy\ x\ 4\emptyset 12 = 266774.4 \text{ N}$$

$$a = \frac{266774.4}{13298.25} = 20.601\text{mm}$$

$$Z = 233.97\ mm$$

$$M_{nf} = TxZ = 62417206.62\ Nmm$$

3 Stress due to P

$$\sigma_p = \frac{62417206.62}{{}^1/_6(150)(300)^2} = 27.74\ {}^N/_{mm^2}$$

4 Stress intensity factor due to P

$$g_1\left(\frac{a}{b}\right) = \frac{1.0-2.5\,a/b+4.49(a/b)^2-3.98(a/b)^3+1.33(a/b)^4}{(1-{}^a/_b)^{3/2}} = 0.982$$

$$K_1^P = \sigma\sqrt{\pi a}\, g_1\left(\frac{a}{b}\right) = 27.74\sqrt{\pi x 100}\ x\ 0.982 = 482.70\ Nmm^{-3/2}$$

5 Wedge forces by reinforcement

x = 56 mm

a = 100 mm

b = 300 mm

$$g_{1\left(\frac{a}{b},\frac{x}{a}\right)} = \frac{3.52(1-{}^x/_a)}{(1-{}^a/_b)^{3/2}} - \frac{4.35-5.28\,{}^x/_a}{(1-{}^a/_b)^{1/2}} + \left[\frac{1.30-0.3({}^x/_a)^{3/2}}{\sqrt{1-({}^x/_a)^2}} + 0.83 - 1.76\,{}^x/_a\right]\left[1-(1-{}^x/_a)\frac{a}{b}\right] = 2.2$$

Due to reinforcement action T = 266774.4 N, results wedge forces p = ${}^{T}/_{150}$

$$K_I^p = \frac{2p}{\sqrt{\pi a}} g_1\left(\frac{a}{b},\frac{x}{a}\right) = 441.613\ \text{Nmm}^{-3/2}$$

Reinforcement contribution to K_I equals to $K_I = K_I^p - K_I^p = (482.70 - 441.613) Nmm^{3/2} = 41.087\ Nmm^{-3/2}$

IV. Discussion

The results of the analysis show that due to 4#12 reinforcement, the crack propagation still occurs at a value of stress intensity factor K_I higher than its critical value (K_I = 41.087 $Nmm^{-3/2}$ and K_{Ic} = 22.13 $Nmm^{-3/2}$). Without reinforcement $K_I = 482.70\ Nmm^{-3/2}$ much greater than K_{Ic} meaning the crack will run very quickly as is common in high quality concrete. Collapse of the structure for this type of material is elastic linear because of unstable crack propagation where the energy absorbed is released instantly. The rate of collapse is also known in terms of the strain energy release rate, also known as fracture energy, G. There are many research in the field of fracture mechanics of concrete showed that the critical of G that is G_{Ic} for normal concrete is about 35 N/m. By applying the relation $K = \sqrt{EG}$, found that the fracture energy G is about 35.17 N/m, a value close to the value for normal strength concrete. This fact implies that brittle collapse in high strength concrete can be diverted to quasi-brittle due to the presence of reinforcement.

V. Conclude

1 Rapid failure of structures can be avoided by creating plastic hinges which is contributed by wedge forces through the composite action between reinforcement and concrete.
2 Yet the micro processes at the local cross section where cracks start to expand (and prevails macro structural failure) can only be accommodated by using the principles of fracture mechanics.
3 Presence of reinforcement, roles an increasing in energy dissipation resulting in increasing ductility as well.

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