International Journal of Engineering Science Invention (IJESI)
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 7 Issue 6 Ver I || June 2018 || PP 14-20

Inventory Ordering Control for a Retrial Service Facility System – Semi- MDP

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Abstract: *We concentrate on the control of ordering inventory in retrial service facility system with (s, S) ordering policy. The arrival of customers to the system is assumed as Poisson and service times are assumed to follows an exponential distribution. For the given values of maximum inventory, maximum waiting space, reorder level and lead times, we determine the optimal ordering policy at various instants of time. The system is formulated as a Semi-Markov Decision Process and the optimum inventory control policy to be employed is found using linear programming (LPP) method. Numerical examples are provided to illustrate the model.*
Keywords *- Single server, Service facility, Retrial queue, Inventory system, Semi-Markov Decision Process*

Date of Submission: 18-05-2018 Date of acceptance: 04-06-2018

I. INTRODUCTION

Most of the communication systems have a queueing situations have the features that customers who find all servers busy upon arrival immediately leave the service area and repeat their request after some random time. This kind of queuing systems with retrials of customers or repeat attempts to obtain service, was originally a topic of telecommunications research. More recently, these systems have served as models for particular computer networks, inventory system and service facilities with inventory. Above examples may explain the current level of activity on the subject. As an example, the "customers" of this queue could be a network of computers attempting to access the same database, which may only be used by one customer at a time. In the case of inventory system customers attempts to get an item from inventory for service completion, if it is not available, he may reattempt from an orbit (virtual space).

In last two decades, many researchers in the field of retrial queuing system, contributed many results. For example, Elcan [8], Arivudainambi et Al. [1], Dragieva [6], Dudin et al.[7] and Artalejo et al. [3, 5] discussed a single server retrial queue with returning customers examined by balking or Bernoulli vacations and derived analytic solutions using Matrix or Generating function or Truncation, methods using level dependent quasi-birth-and-death process (LDQBD).

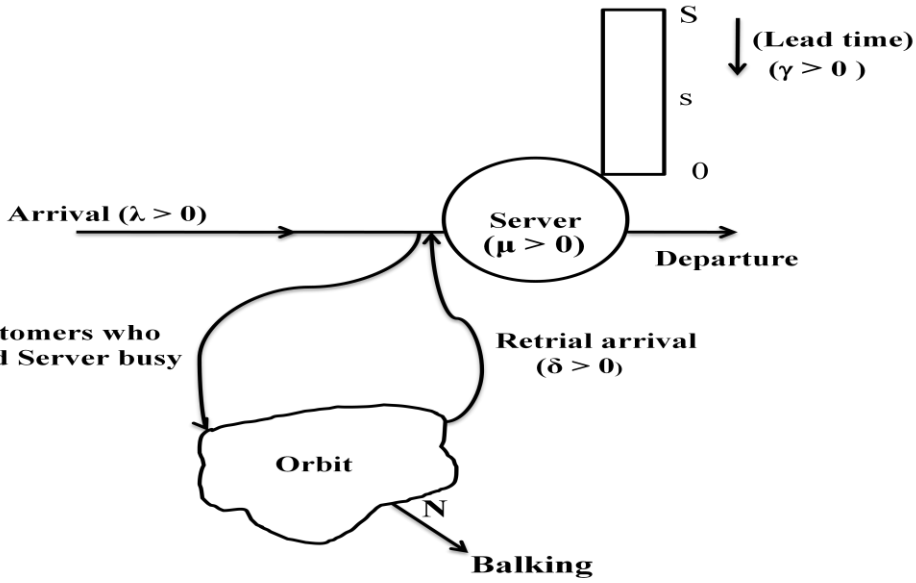
Paul et al. [13] and Krishnamoorthy et al. [11, 12] analyzed a continuous review inventory system at a service facility with retrial of customers. In all these systems, arrival of customers form a Poisson process and service and retrial times are independent and exponentially distributed. They investigate the systems to compute performance measures and construct suitable cost functions for the optimization purpose.

The main contribution of this article is to derive the optimum ordering control rule for the inventory with non – adjusted replenishment process ie., (s, S) policy with $Q = S - s > s$. A retrial service facility system maintaining inventory for service is considered and the orbit is assumed to have finite waiting space. For the given values of maximum waiting space, maximum inventory level, reorder level s and lead times, the system is formulated as a Semi-Markov Decision Process and the optimum inventory policy to be employed is obtained using linear programming method so that the long – run expected cost rate is minimized.

The rest of the article is organized as follows. Preliminary concepts of retrial queues is given in section 1.A brief account of Markov process with continuous time space is described in section 2. We provide a formulation of our Semi - Markov Decision model in the next section 3. In section 4, we present a procedure to prove the existence of a stationary optimal control policy and solve it by employing LP technique.

II. Problem Formulation

Consider a single server service facility system maintaining inventory for service. Customers arrive the system according to a Poisson process with rate $\lambda\ (>0)$. When the server is idle the arriving customer directly enters the server gets service and leaves the system. An arriving customer who finds a server busy is obliged to leave the service area and repeats his request from finite source of a virtual space namely orbit is of finite capacity N. A reattempt made by customer after a random time for the service from the orbit is called retrial. Customer's retrials for service from an orbit follow an exponential distribution with rate δ (> 0). (If there are j customers stay in the orbit the retrial rate is $j\delta$). A customer who sees already N customers in the orbit tend to balk the system (only a finite of demand for the system).



Service times of customers are independent of each other and have a common exponential distribution with parameter μ (> 0). One (unit) of item is served to each customer during service. The maximum capacity of the inventory is fixed as S. Whenever the inventory level reaches to a prefixed level s ($0 \le s < S$), a decision for ordering or non – ordering is taken. Consequently, at levels 1, 2, 3, …, s-1, s decision is taken for ordering or non – ordering and at level 0 compulsory order is placed and the inventory replenishment is non – adjusted. The lead time follows an exponential distribution with parameter γ (> 0). Whenever the inventory level reaches to zero, the arriving customers enter the orbit, status of the server remains 0.

III. Analysis of System

Let X(t), N(t) and I(t) denotes the status of the server, number of customers in the orbit and inventory level at time t, respectively.

Then $\{(X(t), N(t), I(t)): t \ge 0\}$ is a three dimensional continuous time Markov process with state space,

$E = \{0, 1\} \times \{0, 1, 2, \ldots, N\} \times \{0, 1, 2, \ldots, S\}$ where, 0 – idle server, 1 – busy server.

Since, the Markov property holds for the above process at decision epochs t, the infinitesimal generator B of the Markov process has entries of the form $\left(b_{(i,j,k)}^{(l,m,n)}\right)$.

Some of the state transitions with the corresponding rate of transitions are given below:

From state (0, j, k) transitions to the following states are possible:

(i) (1, j, k) with rate λ for $0 \le j \le N$ and $1 \le k \le S$ (primary customer arrival).

(ii) (1, j–1, k) with rate $j\delta$ for $1 \le j \le N$ and $1 \le k \le S$ (Customer arrival from orbit).

From state (1, j-1, k) transitions to the following states are possible:

(i) (1, j, k) with rate λ for $1 \le j \le N$ and $1 \le k \le S$ (primary customer arrival).

(ii) (0, j-1, k-1) with rate μ for $1 \le j \le N$ and $1 \le k \le S$ (Service completion).

From states (0, j, 0) transitions are possible to the states (0, j +1, 0) for $0 \le j \le N - 1$ (primary customer arrival when k = 0).

From states (0, j, k) transitions are possible to the states (0, j, Q + k) for $0 \le j \le N$ and $0 \le k \le s$ (replenishment order Q items is placed).

From states (1, j, k) transitions are possible to the states (1, j, Q + k) for $1 \le j \le N$ and $0 \le k \le s$ (replenishment order Q items is placed).

3.1. MDP formulation

Now, we formulate the infinite planning horizon in by considering the following five components:

1. **Decision epochs:** The decision epochs for the infinite planning horizon system are taken as random points of time say the service completion times.
2. **State space:** Three dimensional space E is considered as the state space.
3. **Action set:** The reordering decisions (0- no order;1- order; 2 –compulsory order) taken at each state of the system (i, j, k) $\in$ E and the replenishment of inventory done at rate γ. The compulsory order for S items is made when inventory level is zero.

Let A_r (r =1, 2, 3) denotes the set of possible actions. Where, $A_1 = \{0\}$, $A_2 = \{0, 1\}$, $A_3 = \{2\}$ and $A = A_1 \cup A_2 \cup A_3$.

The set of all possible actions are:

$$A = \begin{cases} \{0\}, & s+1 \le k \le S \\ \{0,1\}, & 1 \le k \le s \\ \{2\}, & k = 0 \end{cases} , \quad (\mathrm{i,j,k}) \in \mathrm{E}.$$

Suppose the policy f (sequence of decisions) is defined as a function f : E → A, given by f (i, j, k) = {(a): (i, j, k) $\in$E, a$\in$A}.

Let E_1 = {(i, j, k) $\in$ E / f (i, j, k) = 0}.

E_2 = {(i, j, k) $\in$ E / f (i, j, k) = 0 or 1},

E_3 = {(i, j, k) $\in$ E / f (i, j, k) = 2}, then $E = E_1 \cup E_2 \cup E_3$.

4. **Transition probability:** $p_{(i,j,k)}^{(\mathrm{l,m,n})}(a)$ denote the transition probability from state (i, j, k) to state (l, m, n) when decision 'a' is made at state (i, j, k).
5. **Cost:** $C_{(\mathrm{i,j},k)}(a)$ denote the cost occurred in the system when action 'a' is taken at state (i, j, k).

3.2. Steady State Analysis:

Let R denote the stationary policy, which is deterministic time invariant and Markovian Policy (MD). From our assumptions it can be seen that $\{(\mathrm{X(t), N(t), I(t)}) : t \ge 0\}$ is denoted as the controlled process $\{(X^R(t), \mathrm{N}^R(t), \mathrm{I}^R(t)) : t \ge 0\}$ when policy R is adopted. The above process is completely Ergodic, if every stationary policy gives rise to an irreducible Markov Chain. It can be seen that for every stationary policy f, $\{\mathrm{X}^f, \mathrm{N}^f, I^f\}$ is completely Ergodic and also the optimal stationary policy R^* exists, because the state and action spaces are finite.

If d_t is the Markovian deterministic decision rule, the expected reward satisfies the transition probability relations.

$$p_t\left((\mathrm{l,m,n}) \mid (\mathrm{i,j,k}), d_t(\mathrm{i,j,k})\right) = \sum_{a \in A_s} p_t\left((\mathrm{l,m,n}) \mid (\mathrm{i,j,k}), \mathrm{a}\right) p_{\mathrm{d}_t\,(\mathrm{i,j,k})}(a)$$

and $$r_t(\mathrm{i,j,k}), d_t(\mathrm{i,j,k}) = \sum_{a \in A_s} r_t(\mathrm{i,j,k,a}) p_{\mathrm{d}_t\,(\mathrm{i,j,k})}(a).$$

For Deterministic Markovian Policy $\pi \in \Pi^{MD}$, where, Π^{MD} denotes the space of Deterministic Markovian policy. Under this policy Π an action $a \in A$ is chosen with probability $\pi_a(\mathrm{i,j,k})$, whenever the process is in state $(i, j, k) \in E$ .Whenever $\pi_a(\mathrm{i,j,k}) = 0\, or 1$, the stationary Markovian policy Π reduces to a familiar stationary policy.

Then the controlled process $\{(X^R, N^R, I^R)\}$, where, R is the deterministic Markovian policy is a Markov process. Under the policy Π, the expected long run total cost rate is given by

$$C^\pi = h\bar{I}^\pi + c_1\bar{w}^\pi + c_2\alpha_a{}^\pi + \beta\alpha_b{}^\pi + g\alpha_c{}^\pi, . \qquad (1)$$

where,

h – holding cost / unit item / unit time,

c_1 – waiting cost / customer / unit time,

c_2 – reordering cost / order,

β – service cost / customer,

g – balking cost / customer,

$\bar{I}^\pi$ – mean inventory level,

$\bar{w}^\pi$ – mean waiting time in orbit,

α_a^π – reordering rate,

$\alpha_b{}^{\pi}$ – service completion rate,

$\alpha_c{}^{\pi}$ – balking rate.

Our objective here is to find an optimal policy π^* for which $C^{\pi^*} \le C^{\pi}$ for every MD policy in Π^{MD} .

For any fixed MD policy $\pi \in \Pi^{MD}$ and $(i, j, k), (l, m, n) \in E,$ define

$$P_{ijk}^{\pi}(l, m, n, t) = Pr\left\{X^{\pi}(t) = l, N^{\pi}(t) = m, I^{\pi}(t) = n \mid X^{\pi}(0) = i, N^{\pi}(0) = j, I^{\pi}(0) = k\right\}$$

Now $P_{ijk}^{\pi}(l, m, n, t)$ satisfies the Kolmogorov forward differential equation $P'(t) = P(t)Q,$ where, Q is an infinitesimal generator of the Markov process $\{(X^{\pi}(t), N^{\pi}(t), I^{\pi}(t)) : t \ge 0\}$.

For each MD policy Π, we get an irreducible Markov chain with the state space E and actions space A which are finite and $P^{\pi}(l, m, n) = \lim_{t \to \infty} P_{ijk}^{\pi}(l, m, n; t)$ exists and is independent of initial state conditions.

Now the system of equations obtained can be written as follows:

$$(\lambda + j\delta) P^{\pi}(0, j, S) = \gamma . P^{\pi}(0, j, s),\ 0 \le j \le N \tag{2}$$

$$(\lambda + \mu) P^{\pi}(1, 0, Q+k) = \lambda P^{\pi}(0, 0, Q+k) + \delta P^{\pi}(0, 1, Q+k) + \gamma P^{\pi}(1, 0, k),\quad 1 \le k \le s, Q = S - s \tag{3}$$

$$(\lambda + \mu) P^{\pi}(1, j, Q+k) = \lambda \sum_{i=0,1} P^{\pi}(i, j-i, Q+k) + (j+1)\delta P^{\pi}(0, j+1, Q+k) + \gamma P^{\pi}(1, j, k),\quad 1 \le j \le N-1, 1 \le k \le s \tag{4}$$

$$\mu P^{\pi}(1, N, Q+k) = \lambda \sum_{i=0}^{1} P^{\pi}(i, N-i, Q+k) + \gamma P^{\pi}(1, N, k),\quad 1 \le k \le s, \tag{5}$$

$$(\lambda + j\delta) P^{\pi}(0, j, Q+k) = \mu P^{\pi}(1, j, Q+k+1) + \gamma P^{\pi}(0, j, k), 0 \le j \le N, 0 \le k \le s-1 \tag{6}$$

$$(\lambda + j\delta) P^{\pi}(0, j, k) = \mu P^{\pi}(1, j, k+1), 0 \le j \le N, s+1 \le k \le Q-1 \tag{7}$$

$$(\lambda + \mu) P^{\pi}(1, 0, k) = \lambda P^{\pi}(0, 0, k) + \delta P^{\pi}(0, 1, k),\quad s+1 \le k \le Q \tag{8}$$

$$(\lambda + \mu) P^{\pi}(1, j, k) = \lambda \sum_{i=0}^{1} P^{\pi}(i, j-i, k) + (j+1)\delta P^{\pi}(0, j+1, k),\quad 1 \le j \le N-1, s+1 \le k \le Q \tag{9}$$

$$\mu P^{\pi}(1, N, k) = \lambda \sum_{i=0}^{1} P^{\pi}(i, N-i, k),\quad s+1 \le k \le Q \tag{10}$$

$$(\lambda + j\delta + \gamma) P^{\pi}(0, j, k) = \mu P^{\pi}(1, j, k+1), 0 \le j \le N, 1 \le k \le s \tag{11}$$

$$(\lambda + \mu + \gamma) P^{\pi}(1, 0, k) = \lambda P^{\pi}(0, 0, k) + \delta P^{\pi}(0, 1, k),\ 1 \le k \le s \tag{12}$$

$$(\lambda + \mu + \gamma) P^{\pi}(1, j, k) = \lambda \sum_{i=0}^{1} P^{\pi}(i, j-i, k) + (j+1)\delta P^{\pi}(0, j+1, k), 1 \le j \le N-1, 1 \le k \le s \tag{13}$$

$$(\mu + \gamma) P^{\pi}(1, N, k) = \lambda \sum_{i=0}^{1} P^{\pi}(i, N-i, k),\ 1 \le k \le s \tag{14}$$

$$(\lambda + \gamma) P^{\pi}(0, 0, 0) = \mu P^{\pi}(1, 0, 1), \tag{15}$$

$$(\lambda + \gamma) P^{\pi}(0, j, 0) = \mu P^{\pi}(1, j, 1) + \lambda P^{\pi}(0, j-1, 0), 1 \le j \le N-1 \tag{16}$$

$$\gamma . P^{\pi}(0, N, 0) = \lambda P^{\pi}(0, N-1, 0) + \mu P^{\pi}(1, N, 1) \tag{17}$$

Together with the above set of equations, the total probability condition

$$\sum_{(i, j, k) \in E} P^{\pi}(i, j, k) = 1, \tag{18}$$

gives steady state probabilities $\{P^{\pi}(i, j, k), (i, j, k) \in E\}$ uniquely.

3.3. System Performance Measures.

The average inventory level in the system is given by

$$\bar{I}^{\pi} = \sum_{i=0}^{1}\sum_{k=1}^{S} k \sum_{j=0}^{N} P^{\pi}(\mathrm{i,j,k}). \tag{19}$$

Expected number of customers in the orbit is given by

$$\bar{W}^{\pi} = \sum_{i=0}^{1}\sum_{j=1}^{N}\sum_{k=1}^{S} j.P^{\pi}(\mathrm{i,j,k}). \tag{20}$$

The inventory reorder rate is given by

$$\alpha_a^{\pi} = \sum_{j=0}^{N}\sum_{k=1}^{s+1} \mu.P^{\pi}(1,\mathrm{j,k}) . \tag{21}$$

The service completion rate is given by

$$\alpha_b^{\pi} = \mu \sum_{j=0}^{N}\sum_{k=1}^{S} P^{\pi}(1,\mathrm{j,k}) . \tag{22}$$

The expected balking rate is given by

$$\alpha_c^{\pi} = \lambda \sum_{i=0}^{1}\sum_{k=1}^{S} P^{\pi}(\mathrm{i,N,k}) + \lambda P^{\pi}(0,\mathrm{N},0) . \tag{23}$$

Now the long run expected cost rate is given by

$$\begin{aligned} C^{\pi} = {} & h\sum_{i=0}^{1}\sum_{k=1}^{S} k \sum_{j=0}^{N} P^{\pi}(\mathrm{i,j,k}) + c_1 \sum_{i=0}^{1}\sum_{j=1}^{N}\sum_{k=1}^{S} j.P^{\pi}(\mathrm{i,j,k}) \\ & + c_2 \sum_{j=0}^{N}\sum_{k=1}^{s+1} \mu.P^{\pi}(1,\mathrm{j,k}) + \beta\mu \sum_{j=1}^{N}\sum_{k=1}^{S} P^{\pi}(1,\mathrm{j,k}) \\ & + g\left[\lambda \sum_{i=0}^{1}\sum_{k=1}^{S} P^{\pi}(\mathrm{i,N,k}) + \lambda P^{\pi}(0,\mathrm{N},0) \right]. \end{aligned} \tag{24}$$

IV. Linear Programming Problem:

4.1Formulation of LPP

In this section we propose a LPP model within a MDP framework. First, we define the variables, D (i, j, k, a) as a conditional probability expression

D (i, j, k, a) = Pr {decision is ‘a’ | state is (i, j, k)}. -------(25)

Since $0 \le$ D (i, j, k, a) ≤ 1, this is compatible with the deterministic time invariant Markovian policies and the Semi–Markovian decision problem can be formulated as a linear programming problem (LPP).

Hence,

$0 \le$ D (i, j, k, a) ≤ 1 and $\sum_{a\in A=\{0,1,2\}} D(i,\ j,\ k,\ a) = 1,$ i = 0, 1; $0 \le j \le N$; $0 \le k \le S$.

For the reformulation of the MDP as LPP, we define another variable y (i, j, k, a) as follows.

$$y(i,\ j,\ k,\ a) = \mathrm{D}(i,\ j,\ k,\ a)\mathrm{P}^{\pi}(i,\ j,\ k). \tag{26}$$

From the above definition of the transition probabilities,

$$P^{\pi}(\mathrm{i,j,k}) = \sum_{a\in A} y(i,\ j,\ k,\ a), (\mathrm{i,j,k}) \in \mathrm{E}, a \in \mathrm{A} = \{0,1,2\} \tag{27}$$

Expressing $P^{\pi}(\mathrm{i,j,k})$ in terms of $y(\mathrm{i,j,k,a})$, the expected total cost rate function (24) is denoted by C^{π}, and the corresponding LPP is:

Minimize

$$\begin{aligned} C^{\pi} = {} & h \sum_{a\in A=\{0,1,2\}} \sum_{i=0}^{1}\sum_{k=1}^{S} k \sum_{j=0}^{N} y(\mathrm{i,j,k,a}) + c_1 \sum_{a\in A=\{0,1,2\}} \sum_{i=0}^{1}\sum_{j=1}^{N}\sum_{k=1}^{S} j.y(\mathrm{i,j,k,a}) \\ & + c_2 \sum_{a\in A=\{0,1,2\}} \sum_{j=0}^{N}\sum_{k=1}^{s+1} \mu.y(1,\mathrm{j,k,a}) + \beta\mu \sum_{a\in A=\{0,1,2\}} \sum_{j=1}^{N}\sum_{k=1}^{S} y(1,\mathrm{j,k,a}) \\ & + g\left[\lambda \sum_{a\in A=\{0,1,2\}} \sum_{i=0}^{1}\sum_{k=1}^{S} y(\mathrm{i,N,k,a}) + \lambda y(0,\mathrm{N},0,2) \right], \end{aligned} \tag{28}$$

subject to the constraints,

(1) $y(i,j,k,a) \geq 0, (\mathrm{i,j,k}) \in \mathrm{E},\ a \in \mathrm{A}_l,\ l = 0,1,2$

(2) $\sum_{l=0}^{2} \sum_{(\mathrm{i,j,k}) \in \mathrm{E}_l} \sum_{a \in A_l} y(\mathrm{i,j,k,a}) = 1,$

and the balance equations (2) – (18) are obtained by replacing $P^{\Pi}(\mathrm{i,j,k})$ by $\sum_{a \in A} y(\mathrm{i,j,k,a})$.

<u>4.2 Lemma:</u>

The optimal solution of the above Linear Programming Problem yields a deterministic policy.

<u>Proof:</u>

From the equations

$$y(\mathrm{i,j,k,a}) = \mathrm{D(i,j,k,a)}\,\mathrm{P}^{\pi}(\mathrm{i,j,k}) \qquad (29)$$

and

$$P^{\pi}(\mathrm{i,j,k}) = \sum_{a \in A} y(\mathrm{i,j,k,a}),\ (\mathrm{i,j,k}) \in \mathrm{E}. \qquad (30)$$

We have, $$D(\mathrm{i,j,k},a) = \frac{y(\mathrm{i,j,k},a)}{\sum_{a=0}^{2} y(\mathrm{i,j,k},a)} \qquad (31)$$

Since the decision problem is completely ergodic every basic feasible solution to the above linear programming problem has the property that for each $(\mathrm{i,j,k}) \in \mathrm{E}, y(\mathrm{i,j,k},a) > 0$ for exactly one a∈A.

Hence, for each (i, j, k) ∈ E, D (i, j, k, a) is 1for atleast one value of a and zero for all other values of a. Thus, given the amount of inventory on – hand and the number of customers in the orbit, we have to choose the order of inventory for which D(i, j, k, a) is 1. Hence any basic feasible solution of the linear programming yields a deterministic policy ■

V. Numerical illustration and Discussion:

In this section we consider a service facility system maintaining inventory to illustrate the MDP model described in section 4, through numerical examples. We have implemented TORA software to solve LPP by simplex algorithm.

Consider the MDP problem with the following parameters:

Example – 1: S = 3, s = 1, N = 4, λ = 2, μ = 3, γ = 1, δ=3, h = 0.1, c_j = 2j; j = 1, 2, g = 1, β = 2

Inventory ordering policy is (s, S), ie., whenever the inventory reaches level s, on order for Q = S – s > s, items are placed. Lead time is exponential distributed with parameter γ = 1 > 0. The optimum cost for the system implementation is $\mathbf{C}^{\pi^*}$ **= 21.528** and Optimal ordering policy for the proposed system is given in the table:

{(X(t), N(t), I(t))}	(0,0,3)	(0,1,3)	(0,2,3)	(0,3,3)	(1,0,3)	(1,1,3)	(1,2,3)	(1,3,3)
Action	0	0	0	0	0	0	0	0
{(X(t), N(t), I(t))}	(0,0,2)	(0,1,2)	(0,2,2)	(0,3,2)	(1,0,2)	(1,1,2)	(1,2,2)	(1,3,2)
Action	0	0	0	0	0	0	0	0
{(X(t), N(t), I(t))}	(0,0,1)	(0,1,1)	(0,2,1)	(0,3,1)	(1,0,1)	(1,1,1)	(1,2,1)	(1,3,1)
Action	0	0	0	0	1	1	1	0
{(X(t), N(t), I(t))}	(0,0,0)	(0,1,0)	(0,2,0)	(0,3,0)				
Action	2	2	2	2				

ie., The optimum ordering policy for the proposed system is

π* = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 2, 2, 2, 2)

Example – 2: S = 5, s = 2, N = 4, λ = 2, μ = 4, γ = 1, δ=3, h = 0.1, c_j = j; j = 1, 2, g = 1, β = 2

The optimum cost for the system is $\mathbf{C}^{\pi^*}$ **= 7.93393** and Optimal ordering policy for the proposed system is given in the table:

{(X(t), N(t), I(t))}	(0,0,5)	(0,1,5)	(0,2,5)	(0,3,5)	(0,4,5)	(1,0,5)	(1,1,5)	(1,2,5)	(1,3,5)	(1,4,5)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t), N(t), I(t))}	(0,0,4)	(0,1,4)	(0,2,4)	(0,3,4)	(0,4,4)	(1,0,4)	(1,1,4)	(1,2,4)	(1,3,4)	(1,4,4)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t), N(t), I(t))}	(0,0,3)	(0,1,3)	(0,2,3)	(0,3,3)	(0,4,3)	(1,0,3)	(1,1,3)	(1,2,3)	(1,3,3)	(1,4,3)
Action	0	0	0	0	0	0	0	0	0	0
{(X(t), N(t), I(t))}	(0,0,2)	(0,1,2)	(0,2,2)	(0,3,2)	(0,4,2)	(1,0,2)	(1,1,2)	(1,2,2)	(1,3,2)	(1,4,2)
Action	0	0	0	0	0	1	1	1	1	0
{(X(t), N(t), I(t))}	(0,0,1)	(0,1,1)	(0,2,1)	(0,3,1)	(0,4,1)	(1,0,1)	(1,1,1)	(1,2,1)	(1,3,1)	(1,4,1)
Action	0	0	0	0	0	1	1	1	1	0

{(X(t), N(t), I(t))}	(0,0,0)	(0,1,0)	(0,2,0)	(0,3,0)	(0,4,0)					
Action	2	2	2	2	2					

ie., The optimum ordering policy for the proposed system is

π* = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 2, 2, 2, 2, 2)

VI. Conclusion

Analysis of inventory control at service facility is fairly recent system study. In most of previous works optimal parameters for ordering policies or system performance measures are determined. We approached the problem in new style using Semi – Markov Decision Process to control optimally the inventory replenishment. The optimum control policy to be employed is found using linear programming method so that the long–run expected cost rate is minimized. In future we propose to extend this model to multi server-retrial Service Facility system with inventory maintenance.

Acknowledgements

The authors feel humble in acknowledging the HOD of the department of Mathematics, the Principal and Management providing conducive atmosphere in the campus and department to do continuous research in Mathematical Sciences.

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