

A Brief Historical Overview Of the Gaussian Curve: From Abraham De Moivre to Johann Carl Friedrich Gauss

Edel Alexandre Silva Pontes¹

¹Department of Mathematics, Federal Institute of Alagoas, Brazil

Abstract : *If there were only one law of probability to be known, this would be the Gaussian distribution. Faced with this uneasiness, this article intends to discuss about this distribution associated with its graph called the Gaussian curve. Due to the scarcity of texts in the area and the great demand of students and researchers for more information about this distribution, this article aimed to present a material on the history of the Gaussian curve and its relations. In the eighteenth and nineteenth centuries, there were several mathematicians who developed research on the curve, including Abraham de Moivre, Pierre Simon Laplace, Adrien-Marie Legendre, Francis Galton and Johann Carl Friedrich Gauss. Some researchers refer to the Gaussian curve as the "curve of nature itself" because of its versatility and inherent nature in almost everything we find. Virtually all probability distributions were somehow part or originated from the Gaussian distribution. We believe that the work described, the study of the Gaussian curve, its history and applications, is a valuable contribution to the students and researchers of the different areas of science, due to the lack of more detailed research on the subject.*

Keywords - *History of Mathematics, Distribution of Probabilities, Gaussian Curve.*

Date of Submission: 09-06-2018

Date of acceptance: 25-06-2018

I. INTRODUCTION

If there were only one law of probability to be known, this would be the Normal distribution or Gaussian distribution or Laplace-Gauss distribution. After all, who is the creator of this major distribution of probabilities? Faced with this uneasiness, this article intends to discuss about this distribution associated with its graph called the normal curve or Gaussian curve or Laplace-Gauss curve. For reasons of didactic and nomenclature of the text we will adopt Gaussian distribution associated with its Gaussian curve graph.

The history of the Gaussian curve is related to the discovery of probability theory. In the eighteenth and nineteenth centuries, there were several mathematicians who developed research on the curve, including Abraham de Moivre, Pierre Simon Laplace, Adrien-Marie Legendre, Francis Galton and Johann Carl Friedrich Gauss. The Gaussian curve is the most important distribution of probabilities [2] [3] and physical, biological, psychological, social and financial phenomena can be adequately modeled by it.

In the early nineteenth century, mathematicians Laplace and Gauss have two primary tools in Statistics:

(a) The Use of Gaussian Distribution, not only as an approximation of the Binomial Distribution, to describe errors.

(b) For large samples, the use of the Gaussian Distribution as an approximate distribution of the mean - Central Limit Theorem or Laplace Theorem. .

In this way, this article aimed to present a material on the history of the Gaussian curve and its relations, due to the scarcity of texts in the area and the great demand of students and researchers for more information about this distribution.

II. CONTRIBUTIONS OF MAJOR MATHEMATICIANS OF THE 17TH AND 14TH CENTURIES

[5] Abraham de Moivre (Figure 1) was born in France, May 26, 1667. French mathematician exiled in England, several were his contributions to science, among them, the Formula de De Moivre, which relates the complex numbers with trigonometry and , especially the Normal Curve in probability theory. In 1711 he published Philosophical Transactions a work on the laws of chance. In 1725, he used scientific bases and actuarial methods for calculating life insurance.



Figure (1): Abraham De Moivre (1667 – 1754)

In 1733 he published *The doctrine of the odds* an extensive work in several editions containing several problems of probabilities, in this work, Moivre, presents the law of errors or distribution curve. The normal distribution was first introduced by the French mathematician Abraham de Moivre (1667-1754) in an article that was reprinted in the second edition of his book "The doctrine of chance" of 1738. He realized that as the number of events of the currencies increased the binomial distribution approached a smooth curve [4].

For Moivre, large errors are rarer than small errors, that is, the larger the errors, the less frequent they will be and the smaller, the more frequent they will be. Thus, the errors are evenly distributed around the arithmetic mean, the point of greatest peak, forming a curve in the form of a symmetrical bell and falling rapidly to the right and left tails.

Moivre called this curve normal because its mean represents the norm, that is, things should all be like the mean; so that everything that deviates from this average is considered error, where equivalence. Moivre defended this idea under the concept of average or median man, idea that provoked Homeric fights in the history of the normal curve. This idea of the average man implies, for example, that all men should have the same height, the same weight, the same intelligence, etc., that is, they should all be medium; deviations from this rule can be considered as "aberrations" of nature! If you do not introduce philosophical conceptions, this way of thinking of Moivre is very useful and practical to understand what is and what is the normal curve [9].

Moivre produced a great deal of other relevant works. A close friend of Sir Isaac Newton, he was elected in 1697, a member of the Royal Society and subsequently to the Academies of Paris and Berlin. He died in London on November 27, 1754.

[5] Pierre Simon Laplace (Figure 2) was born in France in Beaumonten-Auge on March 23, 1749. Mathematician, astronomer organized mathematical astronomy and published a masterpiece, in five volumes, entitled *Mécanique Céleste*. This work translated the geometric study of classical mechanics used by Isaac Newton for a study based on calculus, known as physical mechanics.



Figure (2): Pierre Simom Laplace (1749 – 1827)

Laplace has developed several research relevant to science: The solution of problems of integral calculus, astronomical mathematics, cosmology, game odds theory, heat theory, metric system, corpuscular optics, sound velocity, among others. In 1783, Laplace employed the normal curve to describe the distribution of errors. In 1812, Laplace published his *Théorie analytique des probabilités*. The method of estimating the proportion of the number of favorable cases, compared to the total number of possible cases, had already been

indicated by Laplace in an article written in 1779. It consists of treating the successive values of any function as coefficients in the expansion of another function, with reference to a different variable.

The latter is therefore called the generating function of the former. Laplace then shows how, by means of interpolation, these coefficients can be determined from the generating function. Then it attacks the problem and, from the coefficients, finds the generating function; this is obtained by the solution of an equation with finite differences. The method is laborious and leads in most cases to a normal distribution of probabilities, the so - called Laplace - Gauss distribution. Laplace was the most influential mathematician of all history in France. In 1806 he became Count of the Empire and was named Marquis in 1817. He died in Paris on March 5, 1827.

[5] Adrien-Marie Legendre (Figure 3) was born in Paris, France on September 18, 1752. In 1770, with only 18 years of age, Legendre defended his doctoral thesis in mathematical physics at Collège Mazarin. Legendre was a great French mathematician, having numerous researches in pure and applied mathematics. He was also an authentic teacher who dedicated himself to basic education. He wrote a book, Elements of Geometry, widely used by students, since the style of his demonstrations was simpler and more accessible. His book was also widely used in Brazil, reaching more than 25 editions.



Figure (3): Adrien-Marie Legendre (1772 – 1833)

In 1782, Legendre was Director of the Berlin Academy of Mathematics. In 1805, Legendre applied the normal curve to introduce the least squares method. Legendre made important contributions to statistics, number theory, geometry, abstract algebra, and mathematical analysis. He died in poverty on January 10, 1833.

[5] Francis Galton, born in Sparkbrook, England on 16 February 1822, was a mathematician, statistician, meteorologist, and anthropologist. It had a very high Intelligence Quotient, around 200.

He was Charles Darwin's cousin and, based on his work, created the concept of "Eugenia" that would be the improvement of a certain species through artificial selection. The first important book for Galton's Psychology was Hereditary Genius (1869). His thesis stated that a remarkable man would have remarkable children. Galton's goal was to encourage the birth of more remarkable or fit individuals in society and discourage the birth of the unfit. He proposed the development of intelligence tests to select bright men and women for selective breeding.

He produced more than three hundred articles and published several books. His main works were: correlation statistics, psychometric, differential psychology, fingerprint, device to open padlocks, geographical distribution of beauty, weight lifting, religious prayer efficiency, intelligence test, teletype printer initial version, and periscope, among others.

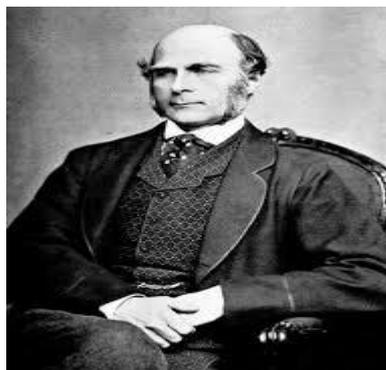


Figure (4): Francis Galton, (1822 – 1911)

In 1872, Galton introduced the name, Gauss Curve. By irony of fate, Gauss was named after the curve, although he had neither created nor named. In 1886, Galton received the Royal Medal, also known as the Medal of the Queen, is a silver medal awarded by the monarch of the United Kingdom, on the recommendation of the Royal Society of London. Galton died on January 17, 1911.

[5] Johann Carl Friedrich Gauss was born in Braunschweig, Germany on April 30, 1777, known as the prince of mathematicians, was also an astronomer and physicist. Considered the greatest mathematician of the time - perhaps of all time - Gauss had an estimated IQ of 240.

One day to keep the class occupied, the teacher had the students add up all the numbers from one to a hundred, with instructions to each one to put his slate on a table as soon as he completed the task. Almost immediately Carl placed his slate on the table, saying, "There it is"; the teacher looked at him with little regard as the others worked diligently. When the master finally looked at the results, the Gaussian slate will see the only one displaying the correct answer, 5050, without any calculations [5].

This fact, Gauss was only ten years old, reports the discovery of the sum of the terms of an arithmetic progression. The professor was so impressed by Gauss's prowess that he paid for arithmetic books for himself. Before that he had learned to read and to add alone.



Figure (5): Johann Carl Friedrich Gauss (1777 – 1855)

In 1809, Gauss published a work, in *Theoria motus*, on the least squares method. Gauss was devoted to mathematics, astronomy, geodesy, physics-mathematics and geometry. In 1823, Gauss published *Theoria combinationis observationum erroribus minimis obnoxiae*, this is the theory of observable errors. In the third section of *Theoria motus*, Gauss introduced the famous law of normal distribution to analyze astronomical data.

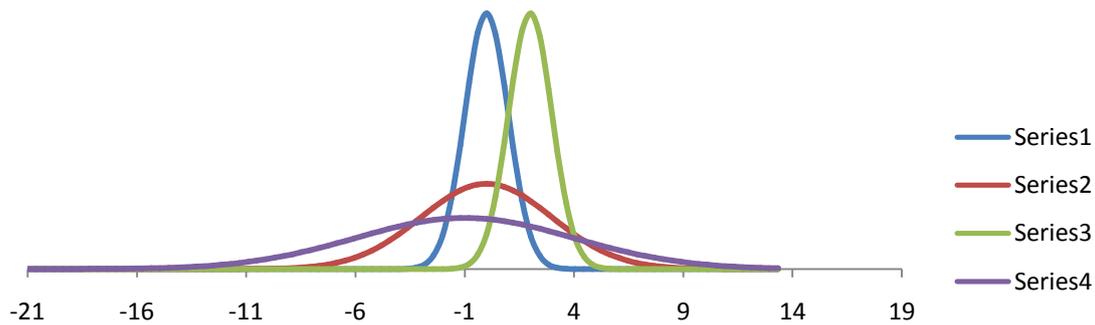
Gauss made a series of general assumptions about observations and observable errors and supplemented them with a purely mathematical assumption. Then, in a very simple way, he was able to obtain the curve equation that corresponded to his empirical results. (...) The graph resembles a bell and is sometimes called a bell-shaped curve. If the coefficient of precision is large, then the curve is steep and the observations fall close to the arithmetic mean. But if it is small, the curve is flat, that is, the distribution is more generalized.

By their publications and their manuscripts they demonstrate that it possessed a differential scientific knowledge. Gauss died on February 23, 1855, in Göttingen, Germany.

III. THE GAUSSIAN CURVE

[7] [10] [11] Let X be a continuous random variable. X follows a Gaussian distribution of arithmetic mean μ and standard deviation σ its probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \text{ onde } x \in (-\infty, +\infty)$$



	Média Aritmética	Desvio Padrão
Série 1	0	1
Série 2	0	3
Série 3	2	1
Série 4	-1	5

FI

Figure (6): Gaussian curves

In this set of graphs, Figure 6, we observe four Gaussian curves with different values for their parameters: Arithmetic mean and standard deviation. The higher the standard deviation the greater the variability of the data around the arithmetic mean. It is also noticed that according to the values of the arithmetic mean and the standard deviation the curve can become flattened or sharper. The platykurtic curve is the flattest curve, short or light tails, remember a plate overturned. The leptokurtic curve is the most elongated curve, long or heavy tails, reminiscent of a mouse. The Mesokurtic curve is our perfect normal curve, not so flat and not so elongated, neutral tails, neither short nor long.

Some researchers refer to the Gaussian curve as the "curve of nature itself" because of its versatility and inherent nature in almost everything we find. Virtually all probability distributions have somehow been part of or originated from the Gaussian distribution. Several are the applications of the Gaussian curve Table 1 shows some phenomena that follows the Gaussian distribution.

1. Duration of human pregnancy	7. Body mass index of athletes
2. Weight of Wistar rats	8. Amount of hemoglobin in men per 100 ml of blood
3. Number of times an adult breathes per minute	9. University student intelligence quotient
4. Height of men and women	10. Salary of employees of a German company
5. Number of rain drops falling in a storm	11. Life of automatic dishwashers
6. Human blood glucose level	12. ENEM Examination Notes

Table (1): Applications of the Gaussian Curve

In Figure 7, it can be seen that 68.26% of the values of a Gaussian random variable are between a positive standard deviation and a negative standard deviation in relation to its arithmetic mean. 95.44% of the values of a Gaussian random variable are between two positive standard deviations and two negative standard deviations relative to their arithmetic mean and 99.74% of the values of a Gaussian random variable are between three positive standard deviations and three standard deviations their average. Values (0.006%) that are spaced three deviations above and three deviations below the mean are called outliers.

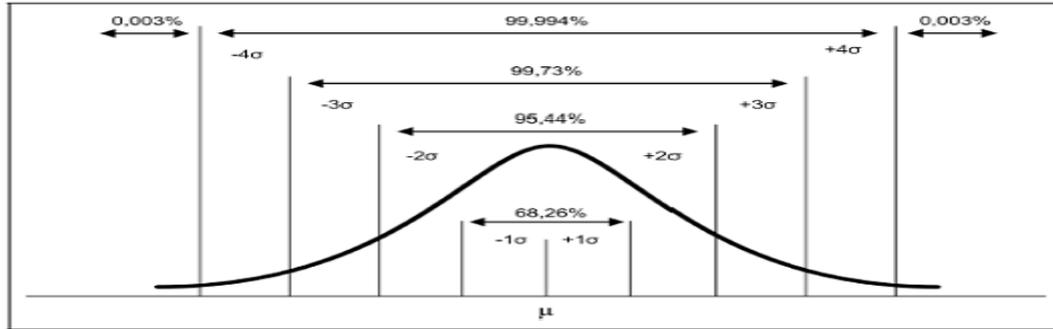


Figure (7): Gaussian Curve Values

The Gaussian curve has several characteristics, in Table 2 we present some of quite relevant.

1.	The Gaussian curve is a continuous random variable X with mean μ and standard deviation σ .
2.	The curve is unimodal and symmetric around the mean, that is, the graph is in the form of a bell.
3.	The highest frequency value, the fashion, coincides with the value of the mean and the median.
4.	The width of the curve is determined by the standard deviation, larger values of standard deviation determine longer and longer curves, showing the variability of the data.
5.	The total area on the curve is one, that is, $\int_{-\infty}^{+\infty} f(x)dx = 1$, where $f(x)$ is the probability density function.
6.	The probability $P(a \leq X \leq b) = \int_a^b f(x)dx$ is the area under the curve in the interval (a, b) .
7.	Most phenomena, in which there is a set of values with a random characteristic, have an approximately normal distribution (following a normal curve).

Table (2): Characteristics of the Gaussian Curve

One reason the Gaussian curve is so important is that for large samples, whatever the distribution of the variable of interest the distribution of sample means will be approximately normal. This result is known as the Central Limit Theorem and allows us to conduct some inferential studies without the need for any knowledge of population distribution.

Central Limit Theorem: Any distribution with sufficiently large sample, from 30 values, has an approximately normal distribution. According to [1], the central limit theorem is of fundamental importance in statistics, it studies the behavior of the sum of random variables, when the number of sums grows, converges to a normal distribution under very general conditions.

The Central Limit Theorem is powerful enough to quantify the uncertainty inherent in statistical inference without making big assumptions that can not be verified [8].

The importance of this distribution lies mainly in the fact that many natural phenomena present a normal or approximately normal distribution. In addition, the means of samples taken from any distribution tend to show normal behavior as the number of observations increases [4].

Most of nature's phenomena follow a regular pattern that arises in sufficiently large groups, though they appear unpredictable and unregular. In two cases we should use the normal curve: (a) when the distribution of the event population is normal, or (b) when the population distribution is not normal but the number of cases is large.

Chama-se curva Gaussiana padronizada, a curva Gaussiana de média $\mu=0$ e desvio padrão $\sigma=1$. As probabilidades da curva normal padrão são facilmente obtidas em tabelas. Em pesquisa quando se fala em curva Gaussiana está assumindo a curva Gaussiana padrão.

[7] [10] [11] Let Z be the standardized Gaussian curve variable of mean $\mu = 0$ and standard deviation $\sigma = 1$. The probability density function is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

The advantage of this standardized curve is that the parameters, mean and standard deviation, are already automatically defined for any measurement scale. To convert the normal curve of mean μ and standard deviation σ to the standard normal curve one must use the formula: $z = \frac{x-\mu}{\sigma}$.

IV. CONCLUSION

The contributions of 18th and 19th century mathematicians in the area of statistics and probability were of fundamental importance for the development of the Gaussian curve. From Moivre to Gauss several were the relevant works in this area. The use of mathematical models is a common practice in the sciences in general.

The Gaussian curve for its properties and characteristics represents the main mathematical model of probability distribution. Understanding the Gaussian curve is an essential requisite for correctly interpreting and

analyzing various phenomena of nature.

We believe that the work described, the study of the Gaussian curve, its history and applications, is a valuable contribution to the students and researchers of the different areas of science, due to the lack of more detailed research on the subject.

REFERENCES

- [1]. Alvarado, Hugo; Batanero, Carmen. Significado del teorema central del límite en textos universitarios de probabilidad y estadística. *Estudios pedagógicos (Valdivia)*, v. 34, n. 2, p. 7-28, 2008.
- [2]. Batanero, Carmen; Tauber, L.; Sanchez, Victoria. Significado y comprensión de la distribución normal en un curso introductorio de análisis de datos. *Cuadrante*, v. 10, n. 1, p. 59-92, 2001.
- [3]. Batanero, Carmen; Tauber, L.; Sanchez, Victoria. Student's reasoning about the normal distribution. In D. Ben-Zvi y J.B. Garfield (Eds) *The Challenge of Developing Statistical Literacy, Reasoning, and Thinking*. Dordrecht: Kluwer. p. 257-76, 2004.
- [4]. Bittencourt, Hélio Radke; Viali, Lori. Contribuições para o ensino da distribuição normal ou curva de Gauss em cursos de graduação. III Simpósio Internacional de Pesquisa em Educação Matemática, 2006.
- [5]. Boyer, Carl, B. *Historia da Matemática*. São Paulo: Edgard Blucher, 1974.
- [6]. Minium, E. W. King, B. M. & Bear, G. (1993). *Statistical Reasoning in Psychology and Education*. John Wiley & Sons, INC.
- [7]. Murray. Chales, & Herrnstein. Richard J. *The Belle Curve intelligence and class structure in American life*. Free Press, 1994.
- [8]. Pagano. M. & Gauvreau. K. *Princípios de Bioestatística*. São Paulo: Pioneira Thomson Learning, 2004.
- [9]. Pasquali, L. *Psicometria - Teoria dos testes na psicologia e na educação*. Petrópolis RJ. Vozes, 2004, p. 158-19
- [10]. Pontes, Edel A. S. Uma Relação Perfeita entre a Curva Gaussiana e o Quociente de Inteligência. *Psicologia & Saberes* v.3. n.4, 2014.
- [11]. Siegal, Sidney. *Estatística não paramétrica para ciências do comportamento*. São Paulo: Mcgraw-Hill do Brasil, 1975.

Edel Alexandre Silva Pontes "A Brief Historical Overview Of the Gaussian Curve: From Abraham De Moivre to Johann Carl Friedrich Gauss "International Journal of Engineering Science Invention (IJESI), vol. 07, no. 06, 2018, pp 28-34