

MHD Convective Flow Past A Vertical Porous Plate Under the Influence of Thermal and Mass Diffusion

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Abstract: In this paper MHD Convective Flow Past A Vertical Porous Plate Under the Influence of Thermal and Mass Diffusion over a vertical porous plate has been examined and the results are presented graphically and physical interpretation is given. The nature of velocity with reference to all critical parameters which appear in the field equations were not commented in detailed. Also, prime importance to the bounding surface with respect to the uniform suction is observed. It is noticed that, as the Prandtl number increases the velocity decreases. Further it is seen that near the boundary layer a backward flow is noticed and subsequently the fluid motion is in the forward direction. Further, it is noticed that as the Prandtl number increases the velocity decreases. Also, it is seen that near the boundary layer a backward flow is noticed and subsequently the fluid motion is in the forward direction. And also, as the Grashoff number increases, the fluid velocity increases. In this case slightly a backward motion is noticed and there after forward motion is observed. Subsequently, it is seen that as the pore size increases the velocity seems to be increasing. It is observed that, as the pore size increases the velocity also increases. In this case more of backward flow is noticed and there after the fluid velocity is dominating as a result of which forward motion is observed. Further, the Prandtl number has the significant contribution over the flow rate.

Keywords-convective flow, mass diffusion, velocity of fluid, flow rate.

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I. Nomenclature

A	Suction parameter
C	Dimensionless species concentration
C_r	Specific heat at constant pressure
C^*	Species concentration
C_w^*	Concentration at the wall
C_∞^*	Concentration in free stream
D	Molecular diffusivity of the species
g	Gravity
Gc	Modified Grashoff number
Gr	Grashoff number
h	Rarefaction parameter
k	Thermal conductivity
K	Dimensionless Permeability parameter
K^*	Permeability parameter
L^*	Constant
M	Magnetic intensity
P_r	Prandtl number
q_w^*	Heat flux at the wall
Sc	Schmidt number
t	Dimensionless time
t^*	Time

T^*	Temperature
T_w^*	Temperature of wall
T_∞^*	Temperature of fluid in free stream
u	Dimensionless velocity component
u^*	Velocity component
V	Suction velocity
V_0^*	Constant mean suction velocity

II. Greek Symbols

- α : Thermal diffusivity
 β : Coefficient of thermal expansion
 β_0 : Coefficient of thermal expansion with concentration
 ϵ : Amplitude ($\ll 1$)
 μ : Viscosity
 ν : Kinematic viscosity
 θ : Dimensionless temperature
 ρ : Density
 σ : Stefan-Boltzmann constant
 τ : Dimensionless shearing stress
 τ^* : Shearing stress
 w : Dimensionless frequency
 w^* : Frequency

III. Introduction

In many of the industrial and environmental cases the radiative convective flow plays a significant role. The applications are more seen in cooling chambers, solar power energy, space technology, energy efficient systems and in cooling systems. The concept is more used in better utilization and design of highly precision equipment which are used in nuclear power plants, propulsion devices in air craft design and in design of gas turbines.

The problem of viscous incompressible fluid on an infinite horizontal plate moving in its own plane was first examined by Stokes [1]. There after the viscous force of a flowing fluid in dense particles was examined by Brinkman [2]. Thereafter, an analytical solution for a viscous flow past a semi-infinite horizontal plate was studied by Stewartson [3]. Subsequently, a two-dimensional steady flow of an incompressible fluid with porous walls, where the flow is influenced by uniform suction or injection was investigated by Berman [4]. Subsequently, Mori [5] studied the flow between two vertical plates which are electrically non-conducting and with the assumption that the wall temperature influences linearly in the direction of the flow. There after the flow in the renal tubes having uniform cross section with a permeable boundary and as the radial velocity varies as exponentially decreasing function was investigated by Macy[6]. Similar such problem by applying finite differences method for the stability of the solution was studied by Hall [7]. Thereafter, Mahajan et al [8] examined the effect of viscous heat dissipating effect in natural convective flows. Later, thermal radiation effects of optionally thin gray gas bounded by a stationary vertical plate was studied by Soundalgekar and Thaker [9]. Subsequently, the radiation effects along a vertical plate with a uniform surface temperature by employing Rossland's approximation was examined by Hossain et al [10]. The effects of thermal radiation and convective flow past a moving infinite vertical plate was presented by Raptis and Perdikis [11]. Subsequently, Antony Raj [12] studied the effects of thermal radiation on the flow past a semi-infinite vertical isothermal plate with uniform heat flux in the presence of transversely applied magnetic field.

In the above studies, the nature of velocity with reference to all critical parameters which appear in the field equations were not commented in detailed. Also, prime importance to the bounding surface with respect to the uniform suction or injection was not studied.

IV. Mathematical Model

An unsteady free convective flow of a viscous incompressible fluid past an infinite vertical porous flat plate in slip-flow regime is examined in this chapter. Additionally, a periodic temperature and concentration when variable suction velocity distribution $[V^* = -V_0^*(1 + \epsilon A e^{i\omega t^*})]$ fluctuating with respect to time is considered. A rectangular Cartesian co-ordinate system with wall lying vertically in x^*y^* -plane is employed. The x^* -axis is taken in vertically upward direction along the vertical porous plate and y^* -axis is taken normal to the plate.

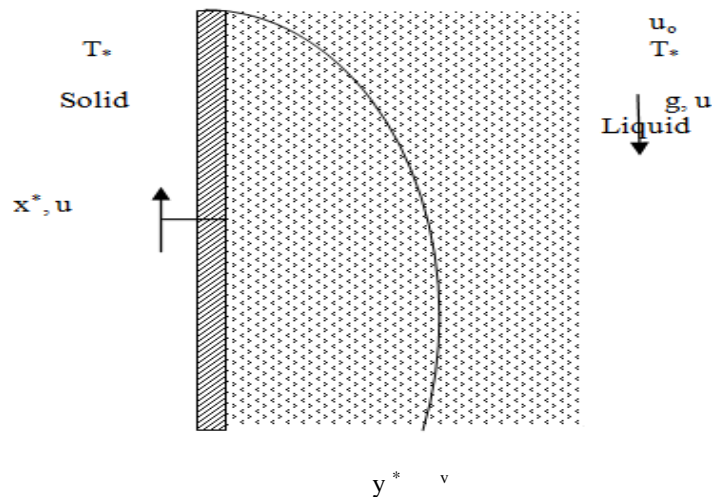


Figure: Schematic representation of the problem

Since the plate is considered infinite in the x^* -direction, hence all physical quantities will be independent of x^* . Under these assumptions, the physical variables are function of y^* and t^* only. Then neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation) the problem can be governed by the following set of equations.

$$\frac{\partial u^*}{\partial t^*} - V_0^*(1 + \varepsilon A e^{i\omega t^*}) \frac{\partial u^*}{\partial y^*} = g\beta(T^* - T_\infty^*) + g\beta^0(C^* - C_\infty^*) + v \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\beta_0^2}{\rho} u^* - \frac{v}{K^*} u^* \quad (1)$$

$$C_p \left[\frac{\partial T^*}{\partial t^*} - V_0^*(1 + \varepsilon A e^{i\omega t^*}) \frac{\partial T^*}{\partial y^*} \right] = k \frac{\partial^2 T^*}{\partial y^{*2}} \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} - V_0^*(1 + \varepsilon A e^{i\omega t^*}) \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \quad (3)$$

The boundary conditions of the problem are:

$$\left. \begin{aligned} u^* &= L^* \left(\frac{\partial u^*}{\partial y^*} \right), T^* = T_w^* + \varepsilon(T_w^* - T_\infty^*) e^{i\omega t^*}, C^* = C_w^* + \varepsilon(C_w^* - C_\infty^*) e^{i\omega t^*} \text{ at } y^* = 0 \\ u^* &\rightarrow 0, T^* \rightarrow T_\infty^*, C^* \rightarrow C_\infty^* \text{ as } y^* \rightarrow \infty \end{aligned} \right\} \quad (4)$$

We now introduce the following non-dimensional quantities into equations

$$y = \frac{y^* V_0^*}{v}, t = \frac{t^* V_0^{*2}}{4v}, u = \frac{4v w^*}{V_0^{*2}}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, Gr = \frac{g\beta v (T_w^* - T_\infty^*)}{V_0^{*3}},$$

$$Gc = \frac{g\beta v (C_w^* - C_\infty^*)}{V_0^{*3}}, Pr = \frac{\mu C_p}{k} = \frac{v \rho C_p}{k}, Sc = \frac{v}{D}, K = \frac{K^* V_0^{*2}}{v^2}, h = \frac{V_0^* L^*}{v}$$

All physical variables are defined in nomenclature where (*) indicates the dimensional quantities. The subscript (∞) denotes the free stream condition. Then equations (1) to (3) reduce to the following non-dimensional form:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = Gr\theta + GcC + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K} \quad (5)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (6)$$

The boundary conditions to the problem in the dimensionless form are:

$$\left. \begin{aligned} u &= h \left(\frac{\partial u}{\partial y} \right), \theta = 1 + \varepsilon e^{i\omega t} \text{ at } y = 0 \\ u &\rightarrow 0, \theta \rightarrow 0, \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (7)$$

V. Method of Solution:

Assuming the small amplitude oscillations ($\varepsilon \ll 1$), we can represent the velocity u , temperature θ and concentration C near the plate as follows:

$$u(y, t) = u_0(y)e^{iwt} \tag{8}$$

$$\theta(y, t) = \theta_0(y)e^{iwt} \tag{9}$$

Substituting (8) to (9) in (5) to (6), equating the coefficients of non-harmonic terms and neglecting of ϵ^2 on both sides

$$u_0'' + u_0' - \left(\frac{1}{K} + \frac{iw}{4}\right)u_0 = -Gr\theta_0 - GcC_0 - Au_0' \tag{10}$$

$$\theta_0'' + Pr\theta_0' - \frac{iwPr}{4}\theta_0 = -A\theta_0'Pr \tag{11}$$

The corresponding boundary conditions reduce to:

$$u_0 = 0, \frac{\partial u_0}{\partial y} = 0 \text{ at } y = 0, \theta_0 = 1, \frac{\partial \theta_0}{\partial y} = 0 \text{ at } y = 0 \tag{12}$$

Where primes denote differentiation with respect to y. Solving the equations (10) and (11) under the boundary conditions (12) we get:

$$u_0(y) = c_1 e^{m_1 y} + c_2 e^{m_2 y} + \frac{Gr\theta_0 + GcC_0}{\left(\frac{1}{K} + \frac{iw}{4}\right)} \tag{13}$$

$$\theta_0(y) = c_3 e^{m_3 y} + c_4 e^{m_4 y} \tag{14}$$

Where

$$m_1 = \frac{-(1+A) + \sqrt{(1+A)^2 + 4\left(\frac{1}{K} + \frac{iw}{4}\right)}}{2}, m_2 = \frac{-(1+A) - \sqrt{(1+A)^2 + 4\left(\frac{1}{K} + \frac{iw}{4}\right)}}{2},$$

$$c_1 = \frac{m_2}{m_1 - m_2} \cdot \frac{Gr\theta_0 + GcC_0}{\left(\frac{1}{K} + \frac{iw}{4}\right)},$$

$$c_2 = \frac{-m_1}{m_1 - m_2} \cdot \frac{Gr\theta_0 + GcC_0}{\left(\frac{1}{K} + \frac{iw}{4}\right)}, m_3 = \frac{-(1+A)Pr + \sqrt{(Pr(1+A))^2 + iwPr}}{2}, m_4 = \frac{-(1+A)Pr - \sqrt{(Pr(1+A))^2 + iwPr}}{2}$$

$$c_3 = \frac{-m_4}{m_3 - m_4}, c_4 = \frac{m_3}{m_3 - m_4}$$

VI. Results and Conclusions:

1. Fig1 and fig2 illustrates the influence of Prandtl number on the velocity profiles for Gr =18, Gr =24 respectively. In each of these illustrations it is noticed that as the Prandtl number increases the velocity decreases. Further, it is seen that near the boundary layer a backward flow is noticed and subsequently the fluid motion is in the forward direction.

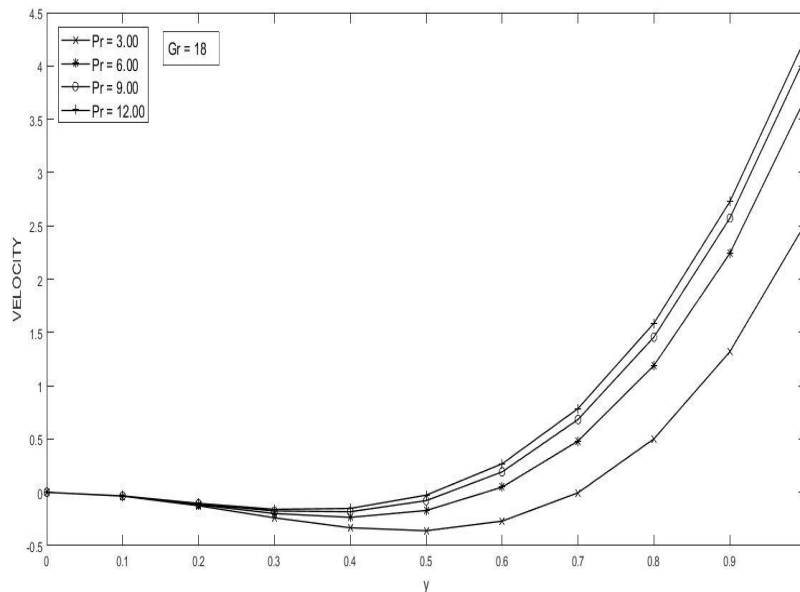


Figure-1: Variation of Velocity profiles w.r.t Prandtl number for Gr = 18

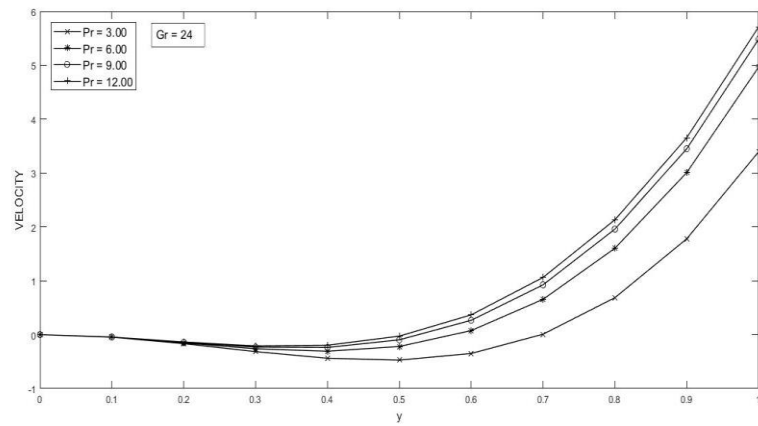


Figure-2: Nature of Velocity profiles for Gr = 24

2. The nature of velocity profiles for a constant Prandtl number is depicted in figure3, figure4 and figure 5. It is noticed that as the Grashoff number increases the fluid velocity increases. In this case also slightly a backward motion is noticed and there after forward motion is observed.

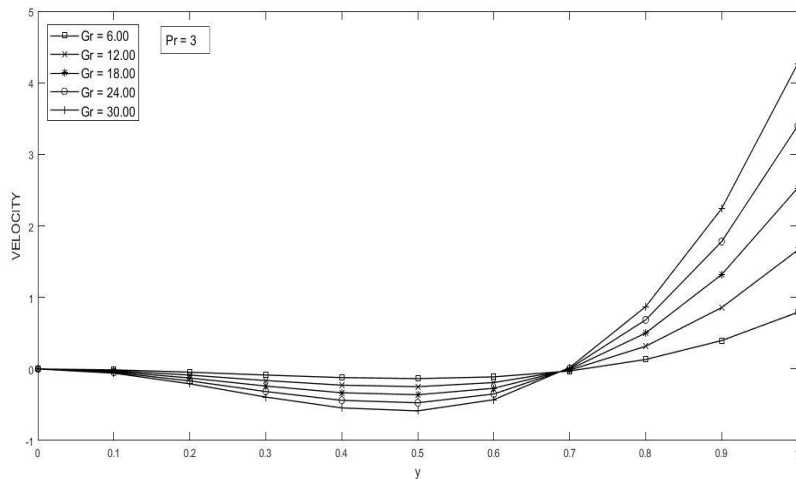


Figure-3: Influence of Grashoff number on Velocity profiles for Pr = 3

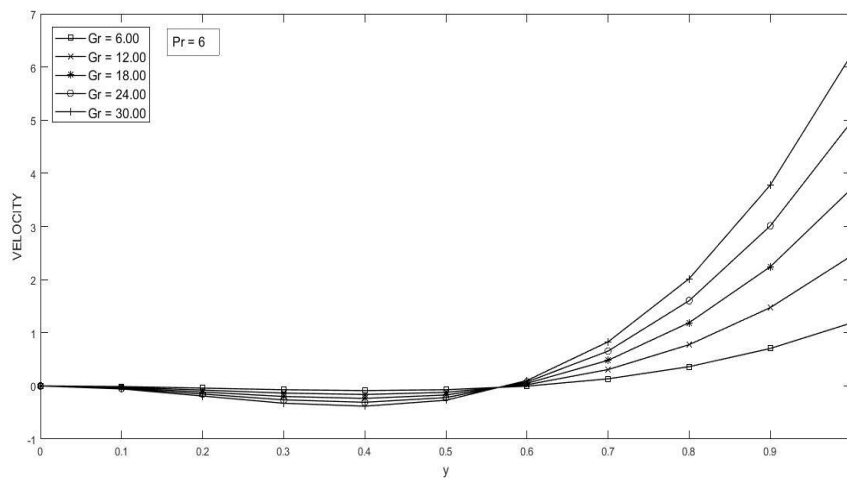


Figure-4: Variation of Velocity profiles w.r.t Grashoff number for Pr = 6

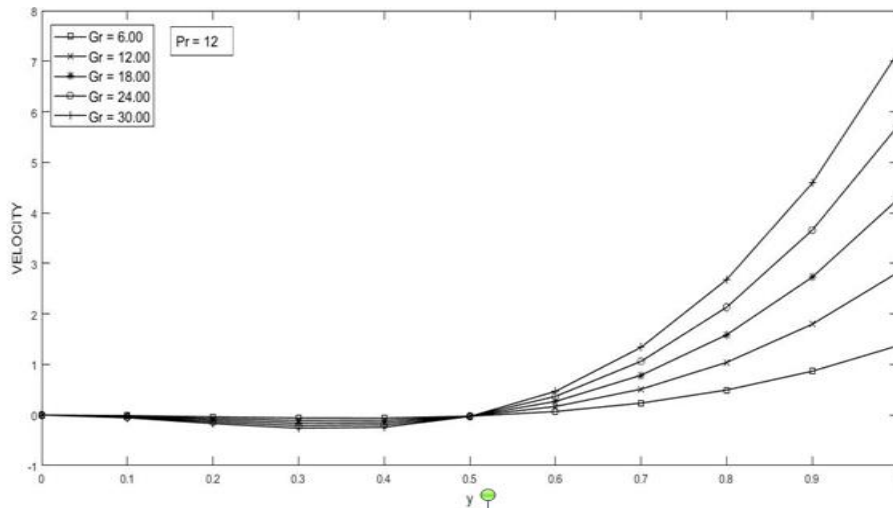


Figure-5: Effect of Grashoff number on Velocity profiles for Pr = 12

- The effect of porosity for a constant Prandtl number is illustrated in figure6. It is seen that as the pore size increases the velocity seems to be increasing.

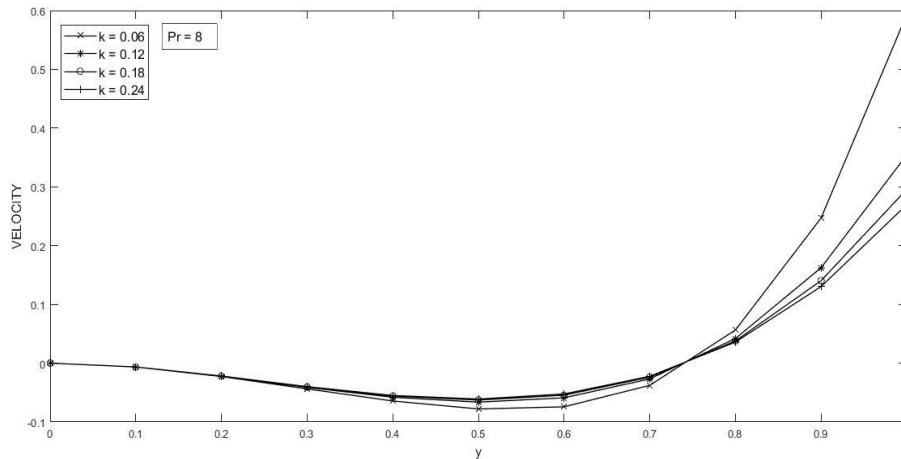


Figure-6: Variation of Velocity profiles w.r.t Porosity for Pr = 8

- Figure 7, figure 8 and figure 9 shows the influence of Prandtl number on the velocity profiles for $k=0.06$, $k=0.12$ and $k=0.24$ respectively. In each of the situations it is observed as the pore size increases the velocity increases. In this case more of backward flow is noticed and there after the fluid velocity is dominating as a result of which forward motion is observed.

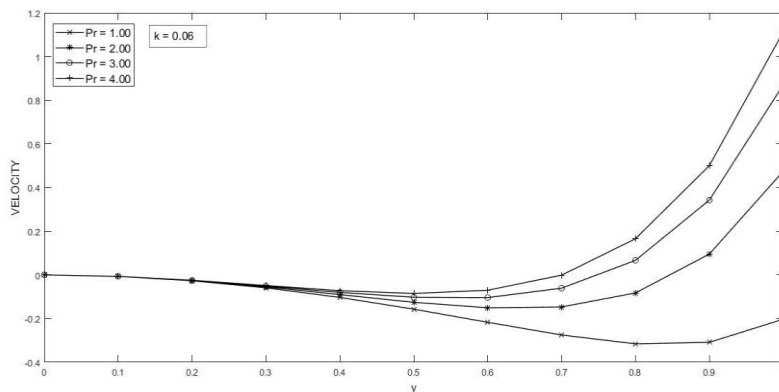


Figure-7: Nature of Velocity profiles for k = 0.06

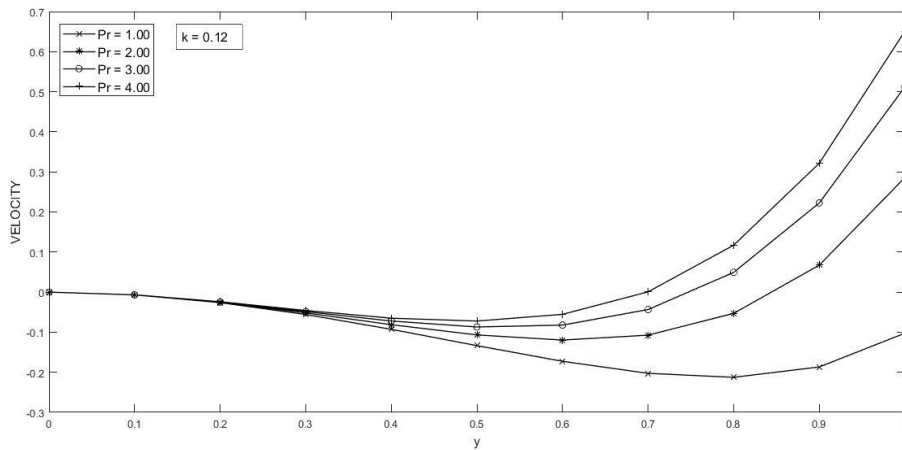


Figure-8: Effect of Prandtl number on Velocity profiles for $k = 0.12$

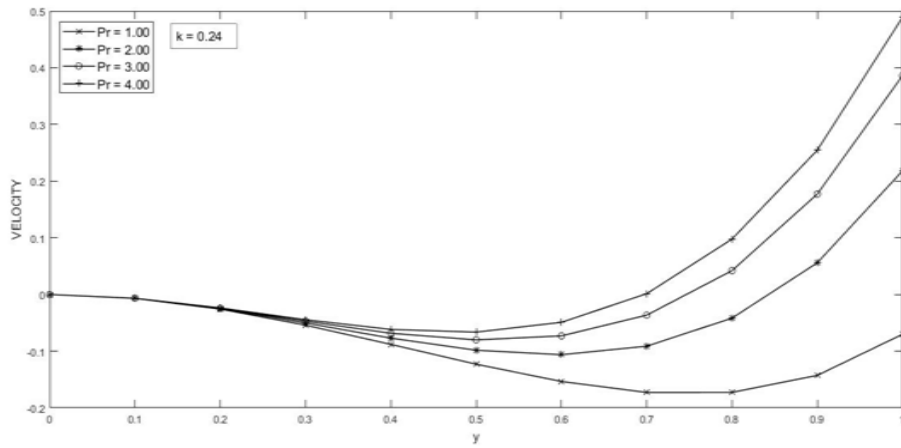


Figure-9: Influence of Prandtl number on Velocity profiles for $k = 0.24$

5. The nature of flow rate for different values of Prandtl number and constant pore size is shown in figure10 and figure11. It is observed that as the Prandtl number increases the flow rate also increases.

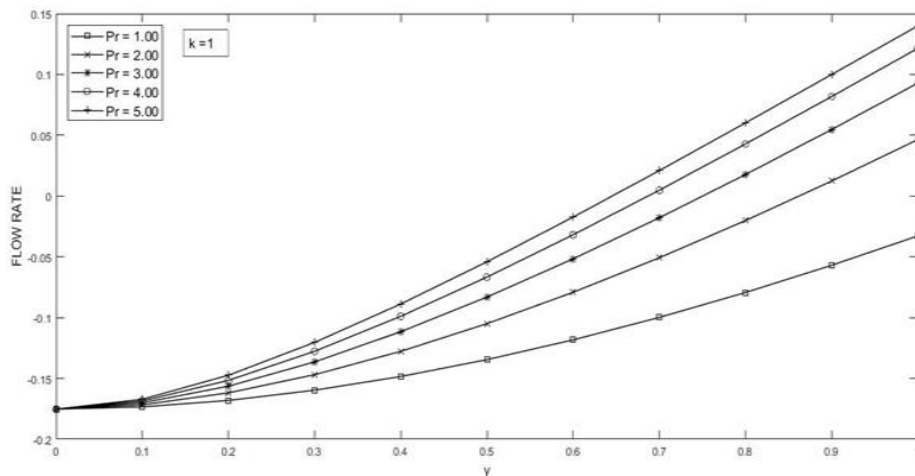


Figure-10: Nature of Flow rate w.r.t Prandtl number for $k = 1$

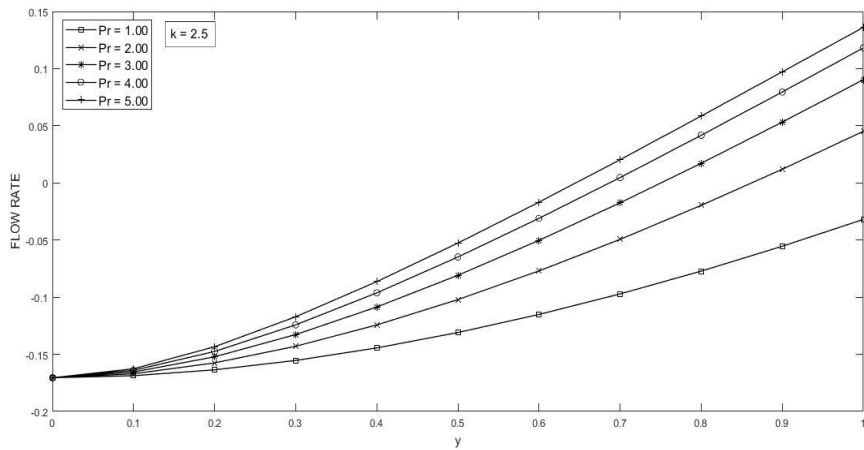


Figure-11: Variation of Flow rate w.r.t Prandtl number for $k = 2.5$

6. Figure 12 and figure 13 depicts the influence of porosity on the flow rate as the porosity of fluid bed changes and the Prandtl number is held constant. In each of these illustrations it is observed that as the porosity increases the flow rate also increases. Further, the Prandtl number has the significant contribution over the flow rate.

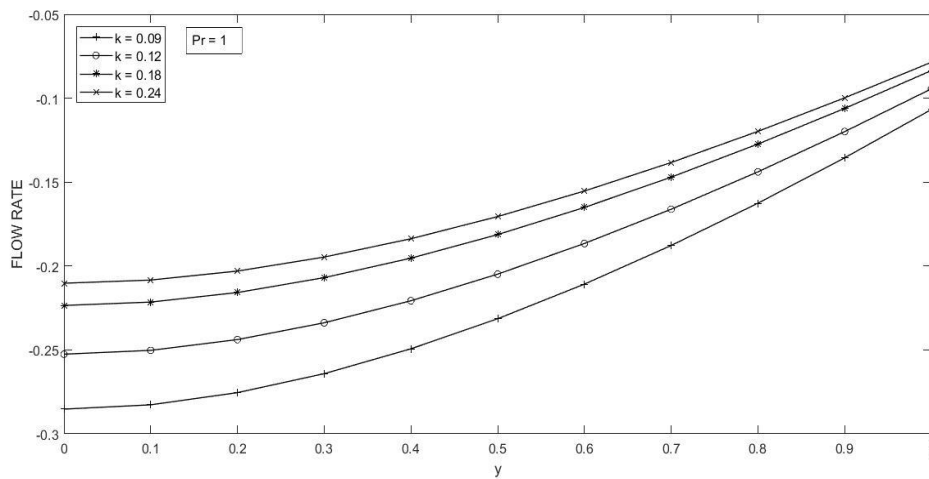


Figure-12: Nature of Flow rate w.r.t Porosity for $Pr = 1$

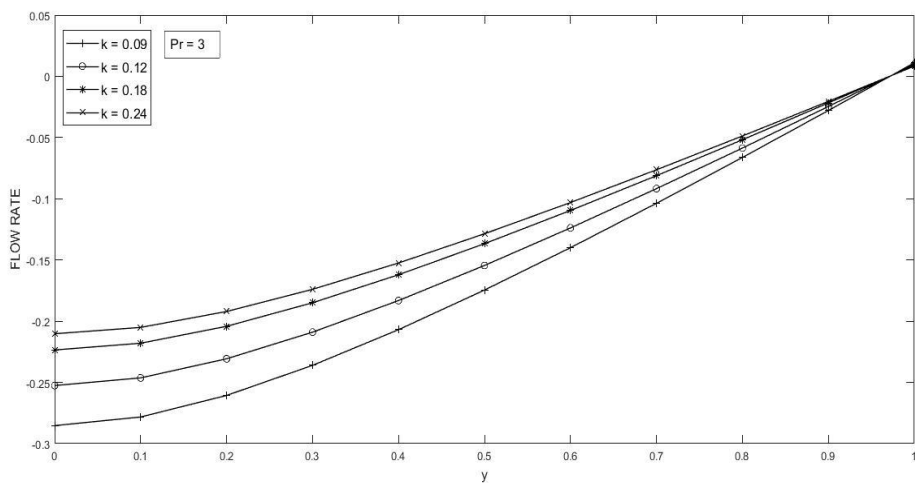


Figure-13: Effect of Porosity on Flow rate for $Pr = 3$

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