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Some Results on Fuzzy δ - Semi Precompactness

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Abstract: *The aim of this paper is to introduce and investigate the concept of a new class of compactness, namely fuzzy δ - semi precompactness in fuzzy topological spaces. Some of their basic properties in fuzzy topological spaces are also to be investigated.*
Key words: *Fuzzy topological space, fuzzy δ - semi preopen set, fuzzy δ - semi preopen cover, fuzzy δ - semi precompactness.*
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I. Introduction

The concept of fuzzy sets was introduced by Prof. L. A. Zadeh [11] in 1965. Realizing the potentiality of introduced notion of fuzzy sets, the researchers successfully applied it for investigations in all the branches of science and technology. In 1968, C. L. Chang [4] introduced the notion of fuzzy topology. S. S. Thakur and S. Singh [9] introduced the concept of fuzzy semi precompactness in fuzzy topological spaces. Also S. Debnath [5] introduced the concept of fuzzy δ - semi compactness. In this paper, the concept of a new kind of fuzzy compactness and some of its basic properties would be investigated in fuzzy setting. Throughout this paper, (X, τ) or simply X will mean a fuzzy topological space (fts, in short) due to Chang [4].

II. Preliminaries

In this section, some known results and definitions are given.

Definition 2.1. A fuzzy subset A of a fuzzy topological space (X, τ) is called

(a) [1] fuzzy semiopen if $A \leq cl(int(A))$,
(b) [2] fuzzy preopen if $A \leq int(cl(A))$,
(c) [8] fuzzy semi preopen if $A \leq cl(int(cl(A)))$,
(d) [6] fuzzy δ - closed if and only if $A = \delta cl(A)$ and the complement of fuzzy δ - closed set is called fuzzy δ - open,
(e) [7] fuzzy δ - semiopen if $A \leq cl(\delta int(A))$,
(f) [3] fuzzy δ - preopen if $A \leq int(\delta cl(A))$,
(g) [10] fuzzy δ - semi preopen if $A \leq \delta cl(int\delta cl(A))$, equivalently, if there exists a fuzzy δ - preopen set B such that $B \leq A \leq \delta cl(B)$. The set of all fuzzy δ - semi preopen sets on X is denoted by FδSPO(X).

Definition 2.2. [4] A fuzzy topological space (X, τ) is fuzzy compact if and only if each fuzzy open covering of X has a finite subcover.

Definition 2.3. [9] A collection $\{B_i : i \in J\}$ of fuzzy preopen sets in a fuzzy topological space (X, τ) is called a fuzzy semi preopen cover of a fuzzy set A of X if $A \leq \vee\{ B_i : i \in J\}$.

Definition 2.4. [9] A fuzzy topological space (X, τ) is said to be fuzzy semi precompact if every fuzzy semi preopen cover of X has a finite subcover.

Definition 2.5. [5] A fuzzy topological space (X, τ) is said to be fuzzy δ - semi compact if every fuzzy δ - semi open covering of X has a finite subcover.

III. Fuzzy δ - semi precompactness

In this section, the concept of fuzzy δ - semi precompactness in fuzzy topological spaces is to be introduced. Also some of its fundamental properties are to be investigated in fuzzy setting.

Definition 3.1. A collection $\{A_i : i \in J\}$ of fuzzy δ - preopen sets in a fuzzy topological space (X, τ) is called a fuzzy δ - semi preopen cover of a fuzzy set B of X if $B \leq \vee \{ A_i : i \in J\}$.

Definition 3.2. A fuzzy topological space (X, τ) is said to be fuzzy δ - semi precompact if every fuzzy δ - semi preopen cover of X has finite subcover.

Remark 3.3. Every fuzzy δ - semiopen cover and fuzzy δ - preopen cover is a fuzzy δ - semi preopen cover. But the converse is not true in general.

Theorem 3.4. Every fuzzy δ - semi precompact space is fuzzy δ - semi compact space.

Proof. Let (X, τ) be a fuzzy δ - semi precompact and let the collection $V = \{A_i : i \in J\}$be a fuzzy δ - semi open cover of X.
Since $\sup_{i \in J}\{B_{Ai}(x) = 1\}$, then $X = \vee A_i$.
By Remark 3.3., the collection V is a fuzzy δ - semi preopen cover of a fuzzy δ - semi precompact space (X, τ).
Therefore, X has a finite subcover which belongs to $V = \{A_i : i \in J\}$.
Hence (X, τ) is a fuzzy δ - semi compact space.

Theorem 3.5. Every fuzzy δ - semi precompact space is fuzzy δ - precompact space.

Proof. Let (X, τ) be a fuzzy δ - semi precompact and let the collection $V = \{A_i : i \in J\}$be a fuzzy δ - preopen cover of X.
Since $\sup_{i \in J}\{B_{Ai}(x) = 1\}$, then $X = \vee A_i$.
By Remark 3.3., the collection V is a fuzzy δ - semi preopen cover of a fuzzy δ - semi precompact space (X, τ).
Therefore, X has a finite sub cover which belongs to $V = \{A_i : i \in J\}$.
Hence (X, τ) be a fuzzy δ - precompact space.

Remark 3.6. A fuzzy δ - semi compact (δ - precompact) space need not be fuzzy δ - semi precompact space.

Theorem 3.7. Every fuzzy δ - semi preclosed subset of a fuzzy δ - semi precompact space is fuzzy compact.

Proof. Let (X, τ) be a fuzzy δ - semi precompact space and F be a fuzzy δ - semi preclosed set of X. It is required to show that F is compact.
Let $V = \{A_i : i \in J\}$ be a fuzzy open cover of F in X.
Since F is a subset of a collection of V, then $\mu_F(x) \leq \sup_{i \in J}\{\mu_{Ai}(x)\}$.
Hence V is a δ - semi preopen cover of F.
F is fuzzy δ - semi preclosed subset of X, F^c is fuzzy δ - semi preopen subset of X.
Therefore, the collection $\{A_i : i \in J\} \cup \{ F^c\}$ is fuzzy δ - semi preopen cover of X which is fuzzy δ - semi precompct space. Then there exists finitely many members of J say, $i_1, i_2, \ldots\ldots, i_n$ such that
$X = \bigcup_{i=1}^{n} A_i \cup \{F^c\}$.
i.e., X has two finite subcovers, say, $\{A_1, A_1, \ldots., A_n\}$ and $\{F^c\}$.
Since $\mu_F(x) \leq 1$, then $F \subseteq X$ and F^c covers no part of X.
Hence $\mu_F(x) \leq \text{Max}\{\mu_{Ai}(x)\}$.
Therefore, $F \subseteq \bigcup_{i=1}^{n} A_i$
Hence F is fuzzy compact.

Theorem 3.8. A fuzzy topological space (X, τ) is fuzzy δ - semi precompact if and only if every family of fuzzy δ - semi preclosed subsets of X with finite intersection property has a non - empty intersection.

Proof.
Necessary part.
Let $F = \{A_i : i \in J\}$be any family of fuzzy δ - semi preclosed subsets of X with finite intersection property. Then the collection $U = \{1_X - A_i : i \in J\}$is a fuzzy δ - semi preopen cover of X. We assert that no finite subfamily of U covers X. For, let $\{1_X - A_i : i = 1, 2,\ldots., n\}$be any non - empty finite subfamily of U. Then $1_X - \vee \{1_X - A_i : i = 1, 2,\ldots., n\} = \wedge \{A_i : i = 1, 2,\ldots., n\} \neq 0_X$ since by hypothesis F has the finite intersection property. This implies that $\vee\{1_X - A_i : i = 1, 2,\ldots., n\} \neq 1_X$. Since X is fuzzy δ - semi precompact it follows that U does not

cover X. Hence $\vee$ {$1_X - A_i$: i $\in$ J} $\neq 1_X$. Consequently, we obtain that $1_X - \wedge$ {A_i : i $\in$ J} $\neq 1_X$. Thus $\wedge$ {A_i : i $\in$ J} $\neq 0_X$.

Sufficient part.
Suppose that every family of fuzzy δ - semi preclosed subset of X with finite intersection property has non - empty intersection. Let {A_i : i $\in$ J}be a fuzzy δ - semi preopen cover of X. Then F = {$1_X - A_i$: i $\in$ J} is a family of fuzzy δ - semi preclosed set in X whose intersection is empty. Therefore, by supposition F does not possess the finite intersection property. And, so there is a finite subfamily say, {$1_X - A_i$: i = 1, 2,…., n} of F with empty intersection. This implies that $\vee$ {A_i : i = 1, 2,….., n} = 1_X. It follows that X is fuzzy δ - semi precompact.

Theorem 3.9. Let (X, τ) be a fuzzy topological space and τ_ζ be a fuzzy topology on X which has FδSPO(X) as a subbase. Then (X, τ) is fuzzy δ - semi precompact if and only if (X, τ_ζ) is fuzzy compact.

Proof. Obvious.

Theorem 3.10. Let (X, τ) be a fuzzy topological space which is fuzzy δ - semi precompact, then each τ_ζ - fuzzy closed set in X is fuzzy δ - semi precompact.

Proof. Let A be any τ_ζ - fuzzy closed set in X and {μ_{Bi} : $B_i \in$ J} be a τ_ζ - fuzzy open cover of A. Since $1_X - A$ is τ_ζ - fuzzy open, {μ_{Bi} : $B_i \in$ J}$\vee$ {$1_X - A$} is a τ_ζ - fuzzy open cover of X. Since by theorem 3.9., X is τ_ζ - compact, there exists a finite subset J_0 of J such that $1_X \leq \vee$ {μ_{Bi}: $B_i \in J_0$} $\vee$ {$1_X - A$}. This implies that A $\leq \vee${μ_{Bi}: $B_i \in J_0$}. Hence A is δ - semi precompact relative to X and this completes the proof.

Theorem 3.11. Let the fuzzy topological space (X, τ) be fuzzy δ - semi precompact. Then every family of τ_ζ - fuzzy closed subsets of X with finite intersection property has non - empty intersection.

Proof. Let X be fuzzy δ - semi precompact. Let F = {μ_{Bi} : $B_i \in$ J}be any family of τ_ζ - fuzzy closed subsets of X with finite intersection property. Suppose $\wedge${ μ_{Bi} : $B_i \in$ J} = 0_X. Then {$1_X - \mu_{Bi}$: $B_i \in$ J}is a τ_ζ - fuzzy open cover of X. Hence it must contain a finite subcover {$1_X - \mu_{Bij}$: j = 1, 2,….., n} for X. This implies that $\wedge${μ_{Bij} : j = 1, 2,….., n} = 0_X which contradicts the assumption that F has a finite intersection property.

Definition 3.12. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let τ_ζ be a fuzzy topology on X which has FδSPO(X) as a subbase. A mapping f : (X, τ) → (Y, σ) is called fuzzy ζ_δ – continuous if f : (X, τ_ζ) → (Y, σ) is fuzzy continuous.

Definition 3.13. Let (X, τ) and (Y, σ) be two fuzzy topological spaces. Let τ_ζ and σ_ζ be respectively the fuzzy topologies on X and Y which have FδSPO(X) and FδSPO(Y) as subbases. A mapping f : (X, τ) → (Y, σ) is called fuzzy $\acute{\zeta}_\delta$ – continuous if f : (X, τ_ζ) → (Y, σ_ζ) is fuzzy continuous.

Theorem 3.14. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let τ_ζ be a fuzzy topology on X which has FδSPO(X) as a subbase. If f : (X, τ) → (Y, σ) is fuzzy δ – semi precontinuous, then f is fuzzy ζ_δ – continuous.

Proof. Let f be fuzzy δ – semi precontinuous and let B $\in$ σ. Then $f^{-1}(B) \in$ FδSPO(X). So $f^{-1}(B) \in \tau_\zeta$. Thus f is fuzzy ζ_δ – continuous.

Theorem 3.15. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let f : (X, τ) → (Y, σ) be fuzzy $\acute{\zeta}_\delta$ – continuous. If a fuzzy subset A of X is fuzzy δ – semi precompact relative to X, then f(A) is fuzzy δ – semi precompact relative to Y.

Proof. Let {μ_{β_i}: $\beta_i \in \Lambda$} be a cover of f(A) by σ_ζ - fuzzy open set in Y. Then {$f^{-1}(\mu_{\beta_i})$: $\beta_i \in \Lambda$} is a cover of A of τ_ζ – fuzzy open set in X and A is fuzzy δ – semi precompact relative to X. Again f is fuzzy $\acute{\zeta}_\delta$ – continuous. So f : (X, τ_ζ) → (Y, σ_ζ) is fuzzy continuous and (X, τ_ζ) is fuzzy compact. Hence A is τ_ζ – fuzzy compact. So there exists a finite subset Λ_0 of Λ such that A $\leq \vee$ {$f^{-1}(\mu_{\beta_i})$: $\beta_i \in \Lambda_0$} and so f(A) $\leq$ {μ_{β_i}: $\beta_i \in \Lambda_0$}. Hence by theorem 3.9., f(A) is τ_ζ – fuzzy compact relative to Y. Thus f(A) is fuzzy δ – semi precompact relative to Y.

Corollary 3.16. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let f : (X, τ) → (Y, σ) be a fuzzy ζ_δ – continuous surjection. If X is fuzzy δ – semi precompact then Y is fuzzy δ – semi precompact.

Proof. Since f is fuzzy ζ_δ – continuous and X is fuzzy δ – semi precompact. So f : (X, τ_ζ) → (Y, σ_ζ) is fuzzy continuous and (X, τ_ζ) is fuzzy compact. This implies that (Y, σ_ζ) is fuzzy compact because f is surjective. Hence by theorem 3.9., Y is fuzzy δ – semi precompact.

Theorem 3.17. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let f : (X, τ) → (Y, σ) be fuzzy ζ_δ – continuous. If a fuzzy subset A of X is fuzzy compact relative to X, then f(A) is fuzzy δ – semi precompact relative to Y.

Proof. Obvious.

Theorem 3.18. Let (X, τ) and (Y, σ) be two fuzzy topological spaces and let f : (X, τ) → (Y, σ) be fuzzy ζ_δ – continuous surjection. If X is fuzzy compact then Y is fuzzy δ – semi precompact.

Proof. Obvious.

Theorem 3.19. Let A and B be two fuzzy subsets of a fuzzy topological space (X, τ) such that A is fuzzy δ – semi precompact relative to X and B is τ_ζ – closed fuzzy set in X. Then A ∧ B is fuzzy δ – semi precompact relative to X.

Proof. Let $\{\mu_{\beta_i} : \beta_i \in \Lambda\}$ be a cover of A ∧ B by τ_ζ – fuzzy subsets of X. Since 1_X – A is τ_ζ – fuzzy open set, $\{\mu_{\beta_i} : \beta_i \in \Lambda\} \vee (1_X - A)$ is fuzzy open cover of A. Since is fuzzy δ – semi precompact, it is τ_ζ – fuzzy compact relative to X. Hence there exists a finite subset Λ_0 of Λ such that $A \leq \vee \{\mu_{\beta_i} : \beta_i \in \Lambda_0\}$. Hence A ∧ B is τ_ζ – fuzzy compact. Therefore A ∧ B is fuzzy δ – semi precompact relative to X.

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